Really Uncertain Business Cycles

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Abstract

We investigate the role of uncertainty in business cycles. First, we demonstrate that microeconomic uncertainty rises sharply during recessions, including during the Great Recession of 2007-2009. Second, we show that uncertainty shocks can generate drops in GDP of around 2.5% in a dynamic stochastic general equilibrium model with heterogeneous firms. However, we also find uncertainty shocks need to be supplemented by first moment shocks to fit consumption over the cycle. So our data and simulations suggest recessions are best modelled as being driven by shocks with a negative first moment and a positive second moment. Finally, we show that increased uncertainty makes first-moment policies, like wage subsidies, temporarily less effective because firms become more cautious in responding to price changes.

Keywords: uncertainty, adjustment costs, business cycles.

JEL Classification: D92, E22, D8, C23.

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1 Introduction

Uncertainty has received substantial attention recently. For example, the Federal Open Market Committee minutes repeatedly emphasize uncertainty as a key factor in the 2001 and 2007-2009 recessions. This paper seeks to evaluate the role of uncertainty for business cycles in two parts. In the first part, we develop new empirical measures of uncertainty using detailed Census microdata from 1972 to 2011, and we highlight three main results. First, the dispersion of plant-level innovations to their total factor productivity (TFP) is strongly countercyclical, rising steeply in recessions. For example, Figure 1 shows the dispersion of TFP shocks for a balanced panel of plants during the two years before the recent recession (2005 to 2006) and two years during the recession (2008 to 2009). Figure 1 shows that plant-level TFP shocks increased in variance by 76% during the recession. Similarly, Figure 2 shows that the dispersion of output growth for these same establishments increased even more, rising by a striking 152% during the recession. Thus, as Figures 1-2 suggest, recessions appear to be characterized by a negative first-moment and a positive second-moment shock to the establishment-level driving processes.

Our second empirical finding is that uncertainty is also strongly countercyclical at the industry level. That is, within SIC 4-digit industries the yearly growth rate of output is negatively correlated with the dispersion of TFP shocks to establishments within the industry. Hence, both at the industry and at the aggregate level, periods of low growth rates of output are also characterized by increased cross-sectional dispersion of TFP shocks.

Our third empirical finding is that for plants owned by publicly traded Compustat parent firms, the size of their plant-level TFP shocks is positively correlated with their parents daily stock returns. Hence, daily stock returns volatility, a popular high-frequency financial measure of uncertainty which also rises in recessions, is tightly linked to the size of yearly plant TFP shocks.

Given this empirical evidence that uncertainty appears to rise sharply in recessions, in the second part of the paper we build a dynamic stochastic general equilibrium (DSGE) model. Various features of the model are specified to conform as closely as possible to the standard frictionless real business cycle (RBC) model as this greatly simplifies comparison with existing work. We deviate from this benchmark in three ways. First, uncertainty is time-varying, so the model includes shocks to both the level of technology (the first moment) and its variance (the second moment) at both the microeconomic and macroeconomic levels. Second, there are heterogeneous firms that are subject to idiosyncratic shocks. Third, the model contains non-convex adjustment costs in both capital and labor. The non-convexities together with time variation in uncertainty imply that firms become more cautious in investing and hiring when uncertainty increases.
The model is numerically solved and estimated using macro and plant level data via a simulated method of moments (SMM) approach. Our SMM parameter estimates suggest that micro and macro uncertainty increase by around threefold during recessions.

Simulations of the model allow us to study its response to an uncertainty shock. Increased uncertainty makes it optimal for firms to wait, leading to significant falls in hiring, investment and output. In our model, overall, uncertainty shocks generate a drop in GDP of around 2.5%. Moreover, the increased uncertainty reduces productivity growth. This reduction occurs because uncertainty reduces the degree of reallocation in the economy since productive plants pause expanding and unproductive plants pause contracting. The importance of reallocation for aggregate productivity growth matches empirical evidence in the U.S. See, for example, Foster, Haltiwanger, and Krizan (2000, 2006), who report that reallocation broadly defined accounts for around 50% of manufacturing and 80% of retail productivity growth in the US.

We then build on our theoretical model to investigate the effects of uncertainty on policy effectiveness. We use a simple illustrative example to show how time-varying uncertainty initially dampens the effect of an expansionary policy. The key to this policy ineffectiveness is that a rise in uncertainty makes firms very cautious in responding to any stimulus.

Our work is related to several strands in the literature. First, we add to the extensive literature building on the RBC framework that studies the role of TFP shocks in causing business cycles. In this literature, recessions are generally caused by large negative technology shocks (e.g. King and Rebelo, 1999). The reliance on negative technology shocks has proven to be controversial, as it suggests that recessions are times of technological regress. As discussed above, our work provides a rationale for at least some portion of variation in measured productivity. Countercyclical increases in uncertainty lead to a freeze in economic activity, substantially lowering productivity growth during recessions.

Second, the paper relates to the literature on investment under uncertainty. A rapidly growing body of work has shown that uncertainty can directly influence firm-level investment and employment in the presence of adjustment costs. Recently, the literature has started to focus on stochastic volatility and its impacts on the economy. Finally, the paper also builds upon a recent literature that studies the role of microeconomic rigidities in general.

equilibrium macro models.\textsuperscript{2}

The remainder of this paper is organized as follows. Section 2 discusses the empirical behavior of uncertainty over the business cycle. In Section 3 we formally present the DSGE model, define the recursive equilibrium, and present our nonlinear solution algorithm. We discuss the estimation of parameters governing the uncertainty process in Section 4, while in Section 5 we study the impact of uncertainty shocks on the aggregate economy. Section 6 studies the implications for government policy in the presence of time-varying uncertainty. Section 7 concludes. Online appendixes include details on the data (A), model solution (B), estimation (C), and a benchmark representative agent model (D).

2 Measuring Uncertainty over the Business Cycle

Before presenting our empirical results, it is useful to briefly discuss what we mean by time-varying uncertainty in the context of our model.

We assume that a firm, indexed by $j$, produces output in period $t$ according to the following production function

$$y_{j,t} = A_t z_{j,t} f(k_{j,t}, n_{j,t}),$$

where $k_{t,j}$ and $n_{t,j}$ denote idiosyncratic capital and labor employed by the firm. Each firm’s productivity is a product of two separate processes: an aggregate component, $A_t$, and an idiosyncratic component, $z_{j,t}$.

We assume that the aggregate and idiosyncratic components of business conditions follow autoregressive processes:

$$\log(A_t) = \rho^A \log(A_{t-1}) + \sigma^A_{t-1} \epsilon_t$$

$$\log(z_{j,t}) = \rho^Z \log(z_{j,t-1}) + \sigma^Z_{t-1} \epsilon_{j,t}.$$  \hfill (2)

We allow the variance of innovations, $\sigma^A_t$ and $\sigma^Z_t$, to move over time according to two-state Markov chains, generating periods of low and high macro and micro uncertainty.

There are two assumptions embedded in this formulation. First, the volatility in the idiosyncratic component, $z_{j,t}$, implies that productivity dispersion across firms is time-varying, while volatility in the aggregate component, $A_t$, implies that all firms are affected by more volatile shocks. Second, given the timing assumption in (2) – (3), firms learn in advance that the distribution of shocks from which they will draw in the next period is changing. This timing assumption captures the notion of uncertainty that firms face about

future business conditions.

These two shocks are driven by different statistics. Volatility in $z_{j,t}$ implies that cross-sectional dispersion-based measures of firm performance (output, sales, stock market returns, etc.) are time-varying, while volatility in $A_t$ induces higher variability in aggregate variables like GDP growth and the S&P500 index. Next we turn to our cross-sectional and macroeconomic uncertainty measures, detailing how both appear to rise in recessions.

2.1 Microeconomic Uncertainty over the Business Cycle

In this section we present a set of results showing that shocks at the establishment-level, firm-level and industry-level all increase in variance during recessions. In our model in Section 3 we focus on units of production, ignoring multi-establishment firms or industry-level shocks to reduce computational burden. Nevertheless, we present data at these three different levels to demonstrate the generality of the increase in idiosyncratic shocks during recessions.

Our first set of measures comes from the Census panel of manufacturing establishments. In summary, with extensive details in Appendix A, this dataset contains detailed output and inputs data on over 50,000 establishments from 1972 to 2011. We focus on the subset of 15,673 establishments with 25+ years of data to ensure that compositional changes do not bias our results, generating a sample of almost half a million establishment-year observations.

To measure uncertainty we first calculate establishment-level TFP ($\tilde{z}_{j,t}$) using the standard approach from Foster, Haltiwanger, and Krizan (2000). We then define TFP shocks ($e_{j,t}$) as the residual from the following first-order autoregressive equation for establishment-level log TFP:

$$\log (\tilde{z}_{j,t}) = \rho \log (\tilde{z}_{j,t-1}) + \mu_j + \lambda_t + e_{j,t},$$

where $\mu_j$ is an establishment-level fixed effect (to control for permanent establishment-level differences) and $\lambda_t$ is a year fixed effect (to control for cyclical shocks). Since this residual also contains plant-level demand shocks that are not controlled for by 4-digit price deflators (see Foster, Haltiwanger and Syverson (2008)) our revenue-based measure will combine both TFP and demand shocks.

Finally, we define microeconomic uncertainty, $\sigma^2_{\tilde{z}^{-1}}$, as the cross-sectional dispersion of $e_{j,t}$ calculated on a yearly basis. In Figure 3 we depict the interquartile range (IQR) of this TFP shock within each year. As Figure 3 shows, the series exhibits a clearly countercyclical behavior. This is particularly striking for the recent Great Recession of 2007-2009, which displays the highest value of TFP dispersion since the series begins in 1972.
Table 1 more systematically evaluates the relationship between the dispersion of TFP shocks and recessions. In column (1) we regress the cross-sectional standard-deviation (S.D.) of establishment TFP shocks on an indicator for the number of quarters in a recession during that year. So, for example, this variable has a value of 0.25 in 2007 as the recession started in quarter IV, and values of 1 and 0.5 in 2008 and 2009, respectively, as the recession continued until quarter II in 2009. We find a coefficient of 0.064 which is highly significant (a t-statistic of 6.9). Given that the mean of the S.D. of establishment TFP shocks is 0.503, a year in recession is associated with a 13% increase in the dispersion of TFP shocks. In the bottom panel we report that this S.D. of establishment TFP shocks also has a highly significant correlation with GDP growth of -0.45.

Our finding here of countercyclical dispersion of micro-level outcomes mirrors a range of other recent papers such as Bachmann and Bayer (2014) in German data, Kehrig (2015) in a similar sample of U.S. Census data, or Jurado, Ludvigson and Ng (2014), Vavra (2014), and Berger and Vavra (2015) for different samples of US firms. A number of these papers build alternative theories or interpretations of such patterns in the microdata qualitatively distinct from our own, but the core empirical regularity of countercyclical micro-level dispersion is remarkably robust.

In columns (2) and (3) we examine the coefficient of skewness and kurtosis of TFP shocks over the cycle and interestingly find no significant correlations. This suggests that recessions can be characterized at the microeconomic level as a negative first-moment shock plus a positive second moment shock. In column (4) we use an outlier-robust measure of cross-sectional dispersion, which is the IQR range of TFP shocks, and again find this rises significantly in recessions. The point estimate on recession of 0.061 implies an increase of over 15% in the IQR of TFP shocks in a recession year. In column (5) as another robustness test we use plant-level output growth, rather than TFP shocks, and find a significant rise in recessions. We also run a range of other experiments on different indicators, measures of TFP, and samples and always find that dispersion rises significantly in recessions.

3This lack of significant correlation was robust in a number of experiments we ran. For example, if we drop the time trend and Census survey year controls the result in column (1) on the standard deviation remains highly significant at 0.062 (0.020), while the results in columns (2) and (3) on skewness and kurtosis remain insignificant at -0.250 (0.243) and -0.771 (2.755). We also experimented with changing the establishment selection rules (keeping those with 2+ or 38+ years rather than 25+ years) and again found the results robust, as shown in Appendix Table A1. Interestingly, Guvenen, Ozkan and Song (2014) find an increase in left-skewness for personal income growth during recessions, which may be absent in plant data because large negative draws lead plants to exit. Because the drop in the left-tail is the key driver of recessions in our model (the “bad news principle” highlighted by Bernanke (1983)), this distinction is relatively unimportant.

4While 15% is a large increase in dispersion it still greatly understates the increase in uncertainty in recession, because a large share of the dispersion of TFP is associated with measurement error. We formally address that in our SMM estimation framework. See Section 4.2 for estimates of the underlying increase in uncertainty in recession and Appendix C for details.

5For example, IQR of employment growth rates has a point estimate (standard error) of 0.051 (0.012), the IQR of TFP shocks measured using an industry-by-industry forecasting equation version of (4) has a point
For example, Figure A1 plots the correlation of plant TFP rankings between consecutive years. This shows that during recessions these rankings churn much more, as increased microeconomic variance leads plants to change their position within their industry-level TFP rankings more rapidly.

In column (6) we use a different dataset which is the sample of all Compustat firms with 25+ years of data. This has the downside of being a much smaller selected sample containing only 2,465 publicly quoted firms, but spanning all sectors of the economy, and providing quarterly sales observations going back to 1962. We find that the quarterly dispersion of sales growth in this Compustat sample is also significantly higher in recessions.

One important caveat when using the variance of productivity ‘shocks’ to measure uncertainty is that the residual $e_{i,t}$ is a productivity shock only in the sense that it is unforecasted by the regression equation (4), rather than unforecasted by the establishment. We address this concern in two ways. First, in column (7) we examine the cross-sectional spread of stock returns, which reflects the volatility of news about firm performance, and again find this is countercyclical, echoing the prior results in Campbell et al. (2001). In fact, as we discuss below in Table 3, we also find that establishment-level shocks to TFP are significantly correlated to their parent’s stock returns, so that at least part of these establishment TFP shocks are new information to the market. Furthermore, to remove the forecastable component of stock returns we repeated the specification in column (7) first removing the quarter by firm mean of firm returns. This controls for any quarterly factors - like size, market/book value, R&D intensity and leverage - that may influence expected stock returns (e.g. Bekaert et al. (2012)), although of course the influence of common factors which may vary at a higher frequency within the quarter may remain. The coefficient (standard error) on recession in these regressions is .019 (0.003), similar to the results obtained in column (7).

Second, we extend the TFP forecast regressions (4) to include additional observables that are likely to be informative about future TFP changes. Adding these in the regression accounts for at least some of the superior information that the establishment might have over the econometrician, helping us in backing out true shocks to TFP from the perspective of the establishments. Figure 4 reports the IQR of the TFP shocks for the baseline forecast regression, as well as for three other dispersion measures, where we sequentially add more variables to the forecasting regressions that are used for recover TFP shocks. First we add two extra lags in levels and polynomials of TFP, next we also include lags and polynomials estimate (standard error) of 0.064 (0.019), using 2+ year samples for the S.D. of TFP shocks we find a point estimate (standard error) of 0.046 (0.014), using a balanced panel of 38+ year establishments we find a point estimate (standard error) of 0.075 (0.015), and using the IQR of TFP shocks measured after removing state-year means, and then applying (4) has a point estimate (standard error) of 0.061 (0.015). Finally, using the IQR of TFP shocks measured after removing firm-year means, and then applying (4) has a point estimate (standard error) of 0.028 (0.011).
of investment, and finally polynomials and lagged in multiple inputs including employment, energy and materials expenditure. As is clear from the figure, even when including forward looking establishment choices for investment and employment, the overall cyclical patterns of uncertainty are almost unchanged.

Finally, in column (8) we examine another measure of uncertainty, which is the cross-sectional spread of industry-level output growth rates, finding again that this is strongly countercyclical.

Hence, in summary plant-level (columns 1–5), firm-level (columns 6–7), and industry-level (column 8) measures of volatility and uncertainty all appear to be strongly countercyclical, suggesting that microeconomic uncertainty rises in recessions.

2.2 Industry Business Cycles and Uncertainty

In Table 2 we report another set of results which disaggregate down to the industry level, finding a very similar result that uncertainty is significantly higher during periods of slower growth. To do this we exploit the size of our Census dataset to examine the dispersion of productivity shocks within each SIC 4-digit industry year cell. The size of the Census dataset means that it has a mean (median) of 27.1 (17) establishments per SIC 4-digit industry-year cell, which enables us to examine the link between within-industry dispersion of establishment TFP shocks and industry growth.

Table 2 displays a series of industry panel regressions in which our dependent variable is the IQR of TFP shocks for all establishments in each industry(y)-year(t) cell. The regression specification that we run is:

\[ IQR_{i,t} = a_i + b_t + \gamma \Delta y_{i,t}. \]

The explanatory variable in column (1) (\(\Delta y_{i,t}\)) is the median growth rate of output between \(t\) and \(t+1\) in the industry-year cell, with a full set of industry \((a_i)\) and year \((b_t)\) fixed effects also included. Column (1) of Table 2 shows that the within-industry dispersion of TFP shocks is significantly higher when that industry is growing more slowly. Since the regression has a full set of year and industry dummies, this is independent of the macroeconomic cycle. So at both the aggregate and industry-level slowdowns in growth are associated with increases in the cross-sectional dispersion of shocks.

This result raises the question of why the within-industry dispersion of shocks is higher during industry slowdowns. In order to explore whether it is the case that industry slowdowns impact some types of establishments differently, we proceed as follows. In columns (2) to (9) we run a series of regressions checking whether the increase in within-industry dispersion is larger given some particular characteristics of the industry. These are regressions
of the form
\[ IQR_{i,t} = a_i + b_t + \gamma \Delta y_{i,t} + \delta \Delta y_{i,t} \times x_i, \]
where \( x_i \) are industry characteristics (see Appendix A for details). Specifically, in column (2) we interact industry growth with the median growth rate in that industry over the full period. The rationale is that perhaps faster growing industries are more countercyclical in their dispersion? We find no relationship, suggesting long-run industry growth rates are not linked to the increase in dispersion of establishment shocks they see in recessions. Similarly, in column (3) we interact industry growth with the dispersion of industry growth rates. Perhaps industries with a wide spread of growth rates across establishments are more countercyclical in their dispersion? Again, we find no relationship. The rest of the table reports similar results for the median and dispersion of plant size within each industry (measured by the number of employees, columns (4) and (5)), the median and dispersion of capital/labor ratios (columns (6) and (7)), and TFP and geographical dispersion interactions (columns (8) and (9)). In all of these we find insignificant coefficients on the interaction of industry growth with industry characteristics.

Thus, to summarize, it appears that: first, the within-industry dispersion of establishment TFP shocks rises sharply when the industry growth rates slow down; and second, perhaps surprisingly, this relationship appears to be broadly robust across all industries.

An obvious question regarding the relationship between uncertainty and the business cycle is the direction of causality. Identifying the direction of causation is important in highlighting the extent to which countercyclical macro and industry uncertainty is a shock driving cycles versus an endogenous mechanism amplifying cycles. A recent literature has suggested a number of mechanisms for uncertainty to increase endogenously in recessions. See, for example, the papers on information collection by Van Nieuwerburgh and Veldkamp (2006) Fagelbaum, Schaal and Tascherau-Dumouchel (2013) or Chamley and Gale (1994), on experimentation in Bachmann and Moscarini (2011), on forecasting by Orlik and Veldkamp (2015), on policy uncertainty by Lubos and Veronesi (2013), and on search by Petrosky-Nadeau andWasmer (2013). Our view is that recessions appear to be initiated by a combination of negative first- and positive second-moment shocks, with ongoing amplification and propagation from uncertainty movements. So the direction of causality likely goes in both directions, and while we model the causal impact of uncertainty in this paper, more work on the reverse (amplification) direction would also be helpful.

2.3 Are Establishment-Level TFP Shocks a Good Proxy for Uncertainty?

The evidence we have provided for countercyclical aggregate and industry-level uncertainty relies heavily on using the dispersion of establishment-level TFP shocks as a measure of
uncertainty. To check this, Table 3 compares our establishment TFP shock measure of uncertainty with other measures of uncertainty, primarily the volatility of daily and monthly firm-stock returns, which have been used commonly in the prior uncertainty literature.\(^6\) Importantly, we note that goal of this section is to demonstrate the correlation between the different measures of uncertainty. Thus, this section does not imply any direction of causation.

In column (1) we regress the mean absolute size of the TFP shock in the plants of publicly traded firms against their parent firm’s within-year volatility of daily stock-returns (plus a full set of firm and year fixed effects). The positive and highly significant coefficient reveals that when plants of publicly quoted firms have large (positive or negative) TFP shocks in any given year, their parent firms are likely to have significantly more volatile daily stock returns over the course of that year. This is reassuring for both our TFP shock measure of uncertainty and stock market volatility measures of uncertainty, as while neither measure is ideal, the fact that they are strongly correlated suggests that they are both proxies for an underlying measure of firm-level uncertainty. In column (2) we use monthly returns rather than daily returns and find similar results, while in column (3) following Leahy and Whited (1996) we leverage adjust the stock returns and again find similar results.\(^7\)

In column (4) we compare instead the within-year standard deviation of firm quarterly sales growth against the absolute size of their establishment TFP shocks. We find again a strikingly significant positive coefficient, showing that firms with a wider dispersion of TFP shocks across their plants tend to have more volatile sales growth within the year. Finally, in column (5) we generate an industry-level measure of output volatility within the year by taking the standard deviation of monthly production growth, and we find that this measure is also correlated with the average absolute size of establishment-level TFP shocks within the industry in that year.

So in summary, establishment-level TFP shocks are larger when the parent firms have more volatile stock returns and sales growth within the year, and the overall industry has more volatile monthly output growth within the year. This suggests these indicators are all picking up some type of common movement in uncertainty.

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\(^6\) See, for example, Leahy and Whited (1996), Schwert (1989), Bloom, Bond, and Van Reenen (2007) and Panousi and Papanikolaou (2012).

\(^7\) As we did in column (7) of Table 1, to remove the forecastable component of stock returns we repeat columns 1 and 3 first removing the quarter by firm mean of firm returns. After doing this the coefficient (standard error) is very similar 0.324 (0.093) for column (1) and 0.387 (0.120) for column (3), mainly because the forecastable component of stock-returns explains a very small of total stock-returns.
2.4 Macroeconomic Measures of Uncertainty

The results discussed so far focus on establishing the countercyclicality of idiosyncratic (establishment, firm, and industry) uncertainty. With respect to macroeconomic uncertainty, existing work has documented that this measure is also countercyclical, including for example Schwert (1989), Campbell, Lettau, Malkiel, and Xu (2001), Engle and Rangel (2008), Jurado, Ludvigsson and Ng (2014), Stock and Watson (2012), or the survey in Bloom (2014).

Rather than repeat this evidence here we simply include one additional empirical measure of aggregate uncertainty, which is the conditional heteroskedasticity of aggregate productivity $A_t$. This is estimated using a $GARCH(1,1)$ estimator on the Basu, Fernald, and Kimball (2006) data on quarterly TFP growth from 1972Q1 to 2010Q4. We find that conditional heteroskedasticity of TFP growth is strongly countercyclical, rising by 25% during recessions which is highly significant (a t-statistic of 6.1), with this series plotted in Appendix Figure A2.

3 The General Equilibrium Model

We proceed by analyzing the quantitative impact of variation in uncertainty within a DSGE model. Specifically, we consider an economy with heterogeneous firms that use capital and labor to produce a final good. Firms that adjust their capital stock and employment incur adjustment costs. As is standard in the RBC literature, firms are subject to an exogenous process for productivity. We assume that the productivity process has an aggregate and an idiosyncratic component. In addition to the standard first-moment shocks considered in the literature, we allow the second moment of the innovations to productivity to vary over time. That is, shocks to productivity can be fairly small in normal times, but become potentially large when uncertainty is high.

3.1 Firms

3.1.1 Technology

The economy is populated by a large number of heterogeneous firms that employ capital and labor to produce a single final good. We assume that each firm operates a diminishing returns to scale production function with capital and labor as the variable inputs. Specifically, a firm indexed by $j$ produces output according to

$$y_{j,t} = A_t z_{j,t} k_{j,t}^{\alpha} n_{j,t}^\nu, \quad \alpha + \nu < 1. \quad (5)$$
Each firm’s productivity is a product of two separate processes: aggregate productivity, $A_t$, and an idiosyncratic component, $z_{j,t}$. Both the macro- and firm-level components of productivity follow autoregressive processes as noted in equations (2) and (3). We allow the variance of innovations to the productivity processes, $\sigma_t^A$ and $\sigma_t^Z$, to vary over time according to a two-state Markov chain.

### 3.1.2 Adjustment Costs

There is a wide literature that estimates labor and capital adjustment costs (e.g. Nickell (1986), Caballero and Engel (1999), Ramey and Shapiro (2001), Hall (2004), Cooper and Haltiwanger (2006), and Merz and Yashiv (2007)). In what follows we incorporate all types of adjustment costs that have been estimated to be statistically significant at the 5% level in Bloom (2009). As is well known in the literature, it is the presence of non-convex adjustment costs that leads to a real options (wait-and-see effect) of uncertainty shocks.

**Capital law of motion**  A firm’s capital stock evolves according to the standard law of motion

$$k_{j,t+1} = (1 - \delta_k)k_{j,t} + i_{j,t}, \quad (6)$$

where $\delta_k$ is the rate of capital depreciation and $i_{j,t}$ denotes investment.

Capital adjustment costs are denoted by $AC_k$ and they equal (i) the sum of a fixed disruption cost $F^K$ for any investment/disinvestment and (ii) a partial irreversibility resale loss for disinvestment (i.e. the resale of capital occurs at a price that is only a share $(1 - S)$ of its purchase price). Formally,

$$AC_k = \mathbb{I}(|i| > 0)g(z, A, k, n)F^K + S|i|\mathbb{I}(i < 0) \quad (7)$$

**Hours law of motion** The law of motion for hours worked is governed by

$$n_{j,t} = (1 - \delta_n)n_{j,t-1} + s_{j,t}, \quad (8)$$

where $s_{j,t}$ denotes the net flows into hours worked and $\delta_n$ denotes the exogenous destruction rate of hours worked (due to factors such as retirement, illness, or exogenous quits, etc...).

Labor adjustment costs are denoted by $AC_n$ in total and they equal (i) the sum of a fixed disruption cost $F^L$ and (ii) a linear hiring/firing cost, which is expressed as a fraction of the aggregate wage ($Hw$). Formally,

$$AC_n = \mathbb{I}(|s| > 0)g(z, A, k, n)F^L + |s|Hw \quad (9)$$

Note that these adjustment costs in labor imply that $n_{j,t-1}$ is a state variable for the firm.
3.1.3 The Firm’s Value Function

We denote by $V(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)$ the value function of a firm. The seven state variables are given by (1) a firm’s capital stock, $k$, (2) a firm’s hours stock from the previous period, $n_{-1}$, (3) the firm’s idiosyncratic productivity, $z$, (4) aggregate productivity, $A$, (5) the current value of macro uncertainty, $\sigma^A$, (6) the current value of micro uncertainty uncertainty, $\sigma^Z$, and, (7) the joint distribution of idiosyncratic productivity and firm-level capital stocks and hours worked in the last period, $\mu$, which is defined for the space $S = R_+ \times R_+ \times R_+$.

Denoting by the primes the value of next period variables, the dynamic problem of the firm consists of choosing investment and hours to maximize

$$V(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) = \max_{i,n} \left\{ y - w(A, \sigma^A, \sigma^Z, \mu)n - i ight.$$ 
$$\left. - AC^k(k, n_{-1}, z, k^A, \sigma^Z, \mu) - AC^n(k, n_{-1}, z, n; A, \sigma^A, \sigma^Z, \mu) \right\}$$

$$+ \mathbb{E} \left[ m(A, \sigma^A, \sigma^Z, \mu; A', \sigma^{Z'}, \mu') V(k', n', z'; A', \sigma^{Z'}, \mu') \right]$$

given a law of motion for the joint distribution of idiosyncratic productivity, capital, and hours,

$$\mu' = \Gamma(A, \sigma^A, \sigma^Z, \mu),$$

and the stochastic discount factor, $m$, which we discuss below in Section 3.4. $w(A, \sigma^A, \sigma^Z, \mu)$ denotes the wage rate in the economy. $K(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)$ and $N^d(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)$ denote the policy rules associated with the firm’s choice of capital for the next period and current demand for hours worked.

3.2 Households

The economy is populated by a large number of identical households that we normalize to a measure one. Households choose paths of consumption, labor supply, and investment in firm shares to maximize lifetime utility. We use the measure $\psi$ to denote the one-period purchased shares in firms. The dynamic problem of the household is given by

$$W(A, \sigma^A, \sigma^Z, \mu) = \max_{\{C,N,\psi\}} \left\{ U(C, N) + \beta \mathbb{E} \left[ W(A', \sigma^{A'}, \sigma^{Z'}, \mu') \right] \right\},$$
subject to the law of motion for $\mu$ and a sequential budget constraint

\[
C + \int q(k', n, z; A, \sigma^A, \sigma^Z, \mu) d\psi'(k', n, z)
\leq w(A, \sigma^A, \sigma^Z, \mu) + \int \rho(k, n-1, z; A, \sigma^A, \sigma^Z, \mu) d\mu(k, n-1, z).
\]

(13)

Households receive labor income as well as the sum of dividends and the resale value of their investments priced at $\rho(k, n-1, z; A, \sigma^A, \sigma^Z, \mu)$. With these resources the household consumes and buys new shares at a price $q(k', n, z; A, \sigma^A, \sigma^Z, \mu)$ per share of the different firms in the economy. We denote by $C(\psi, A, \sigma^A, \sigma^Z, \mu)$, $N^s(\psi, A, \sigma^A, \sigma^Z, \mu)$, and $\Psi'(k', n, z; A, \sigma^A, \sigma^Z, \mu)$ the policy rules determining current consumption, time worked, and quantities of shares purchased in firms that begin the next period with a capital stock $k'$ and who currently employ $n$ hours with idiosyncratic productivity $z$.

### 3.3 Recursive Competitive Equilibrium

A recursive competitive equilibrium in this economy is defined by a set of quantity functions $\{C, N^s, \Psi, K, N^d\}$, pricing functions $\{w, q, \rho, m\}$, and lifetime utility and value functions $\{W, V\}$. $V$ and $\{K, N^d\}$ are the value and policy functions solving (10) while $W$ and $\{C, N^s, \Psi'\}$ are the value and policy functions solving (12). There is market clearing in asset markets

\[
\mu'(k', n, z') = \int \psi'(k', n, z)f(z'|z)dz,
\]

the goods market

\[
C(\psi, A, \sigma^A, \sigma^Z, \mu)
= \int \left[ \begin{array}{c}
Az^\alpha N^d(k, n-1, z; A, \sigma^A, \sigma^Z, \mu) - (K(k, n-1, z; A, \sigma^A, \sigma^Z, \mu) - (1 - \delta_k)k)

-AC^k(k, K(k, n-1, z; A, \sigma^A, \sigma^Z, \mu)) - AC^a(n-1, N(k, n-1, z; A, \sigma^A, \sigma^Z, \mu))
\end{array} \right] d\mu(k, n-1, z),
\]

and the labor market

\[
N^s(\psi, A, \sigma^A, \sigma^Z, \mu) = \int N^d(k, n-1, z; A, \sigma^A, \sigma^Z, \mu) d\mu(k, n-1, z).
\]

Finally, the evolution of the joint distribution of $z, k$ and $n-1$ is consistent. That is, $\Gamma(A, \sigma^A, \sigma^Z, \mu)$ is generated by $K(k, n-1, z; A, \sigma^A, \sigma^Z, \mu)$, $N^d(k, n-1, z; A, \sigma^A, \sigma^Z, \mu)$, and the exogenous stochastic evolution of $A, z, \sigma^Z$ and $\sigma^A$, along with the appropriate integration of firms’ optimal choices of capital and hours worked given current state variables.
3.4 Sketch of the Numerical Solution

We briefly describe the solution algorithm, which heavily relies on the approach in Khan and Thomas (2008) and Bachmann, Caballero and Engel (2013). Fuller details are laid out in Appendix B, and the full code is available online.

The model can be simplified substantially if we combine the firm and household problems into a single dynamic optimization problem. From the household problem we get

\[ w = - \frac{U_N(C, N)}{U_C(C, N)}, \quad (14) \]

\[ m = \beta \frac{U_C(C', N')}{U_C(C, N)}, \quad (15) \]

where equation (14) is the standard optimality condition for labor supply and equation (15) is the standard expression for the stochastic discount factor. We assume that the momentary utility function for the household is separable across consumption and hours worked,

\[ U(C_t, N_t) = C_t^{1-\eta} (1-\eta) - \theta^\chi N_t^\chi, \quad (16) \]

implying that the wage rate is given by

\[ w_t = \theta N_t^{\chi-1} C_t^\eta. \quad (17) \]

We define the intertemporal price of consumption goods as \( p(A, \sigma^Z, \sigma^A, \mu) \equiv U_C(C, N) \). This then allows us to redefine the firm’s problem in terms of marginal utility, denoting the new value function as \( \tilde{V} \equiv pV \). The firm problem can then be expressed as

\[ \tilde{V}(k, n, z; A, \sigma^A, \sigma^Z, \mu) = \max_{i,n} \left\{ p(A, \sigma^A, \sigma^Z, \mu) \left( y - w(A, \sigma^A, \sigma^Z, \mu)n - i - AC^k - AC^n \right) + \beta \mathbb{E} \left[ \tilde{V}(k', n', z', A', \sigma^{A'}, \sigma^{Z'}, \mu') \right] \right\}. \quad (18) \]

To solve this problem we employ nonlinear techniques that build upon Krusell and Smith (1998). Detailed discussion of the algorithm is provided in Appendix B where we implement a range of alternative implementations of our Krusell-Smith type algorithm. Importantly, as we discuss in Appendix B the main results remain robust across the different alternatives we consider.
4 Parameter Values

In this section, we describe the quantitative specification of our model. To maintain comparability with the RBC literature, we perform a standard calibration when possible (see Section 4.1 and Table 4). However, the parameters that govern the uncertainty process can neither be calibrated to match first moments in the U.S. data, nor have they been previously estimated in the literature. As such, we adopt a simulated method of moments (SMM) estimation procedure to choose these values in Section 4.2. In Section 5.2.3, we explore the sensitivity of the results to different parameter values.

4.1 Calibration

Frequency and Preferences We set the time period to equal a quarter. The household’s discount rate, $\beta$, is set to match an annual interest rate of 5%. $\eta$ is set equal to one which implies that the momentary utility function features an elasticity of intertemporal substitution of one (i.e. log preferences in consumption). Following Khan and Thomas (2008) and Bachmann, Caballero and Engel (2013) we assume that $\chi = 1$. This assumption implies that we do not need to forecast the wage rate in addition to the forecast of $p$ in our Krusell-Smith algorithm, since the household’s labor optimality condition with $\chi = 1$ implies that the wage is a function of $p$ alone. We set the parameter $\theta$ such that households spend about a third of their time working in the non-stochastic steady state.

Production Function, Depreciation, and Adjustment Costs We set $\delta_k$ to match a 10% annual capital depreciation rate. Based on Shimer (2005) we set the annual exogenous quit rate to 35%. We set the exponents on capital and labor in the firm’s production function to be $\alpha = 0.25$ and $\nu = 0.5$, consistent with a capital cost share of $1/3$ of total input costs.

As previously discussed, the existing literature provides a wide range of estimates for capital and labor adjustment costs. We use adjustment cost parameters from Bloom (2009). The resale loss of capital amounts to 34%. The fixed cost of adjusting hours is set to 2.1% of annual sales, and the hiring and firing costs equal 1.8% of annual wages.

Aggregate and Idiosyncratic TFP Processes Productivity, both at the aggregate and the idiosyncratic level, is determined by AR(1) processes as specified in equations (2) and (3). The serial autocorrelation parameters $\rho^A$ and $\rho^Z$ are set to 0.95, similar to the quarterly value used by Khan and Thomas (2008).
4.2 Estimation

The Uncertainty Process  We assume that the stochastic volatility processes, $\sigma^A_t$ and $\sigma^Z_t$, follow a two-point Markov chain:

$$\sigma^A_t \in \{\sigma^A_L, \sigma^A_H\} \quad \text{where} \quad Pr(\sigma^A_{t+1} = \sigma^A_j | \sigma^A_t = \sigma^A_k) = \pi^A_{k,j}$$

$$\sigma^Z_t \in \{\sigma^Z_L, \sigma^Z_H\} \quad \text{where} \quad Pr(\sigma^Z_{t+1} = \sigma^Z_j | \sigma^Z_t = \sigma^Z_k) = \pi^Z_{k,j}. \number{19}$$

Since we cannot directly observe the stochastic process of uncertainty in the data we proceed with SMM estimation. We formally discuss in Appendix C the estimation procedure and all relevant details.

Since the empirical results in Section 2 suggested that microeconomic and macroeconomic uncertainty move through the business cycle, we assume that a single process determines the economy’s uncertainty regime. That is, our assumption of a single uncertainty process implies that whenever microeconomic uncertainty is low (or high) so is macroeconomic uncertainty. This assumption reduces the number of parameters governing the uncertainty process to the following six: $\sigma^A_L, \sigma^A_H, \sigma^Z_L, \sigma^Z_H, \pi^A_{H,L}$ and $\pi^A_{L,H}$.

As the uncertainty process has a direct impact on observable cross-sectional and aggregate time series moments, it is natural that the SMM estimator minimizes the sum of squared percentage deviations of the following eight model and U.S. data moments: At the microeconomic level, we target the (i) mean, (ii) standard deviation, (iii) skewness, and (iv) autocorrelation of the time series of the cross-sectional interquartile range of establishment TFP shocks computed from our annual Census sample covering 1972-2010. At the macro level, we target the same four moments based on the time series of estimated heteroskedasticity using a GARCH(1,1) model for the annualized quarterly growth rate of the U.S. Solow residual, covering 1972Q1-2010Q4. We display the estimated uncertainty process parameters in Table 5 and the targeted moments in Table 6.

Based on this estimation procedure we find that periods of high uncertainty occur with a quarterly probability of 2.6%. The period of heightened uncertainty is estimated to be persistent with a quarterly probability of 94% of staying in the high uncertainty state. Aggregate volatility is 0.67% with low uncertainty and increases by approximately 60% when an uncertainty shock arrives. Idiosyncratic volatility is estimated to equal 5.1% and increases by approximately 310% in the heightened uncertainty state. Table 5 reports the point estimates and standard errors of from the SMM estimation procedure. As the table shows, most of these parameters are estimated precisely. However, in Section 5.2.3 we discuss the robustness of our numerical results to modification of each of these six parameters.

It is useful at this point to explain the large estimated increase in underlying fundamental
microeconomic uncertainty $\sigma_t^Z$ on impact of an uncertainty shock in light of the apparently more muted fluctuations of our microeconomic uncertainty proxy in Figure 3. Although closely related and informative for one another, the two series are distinct. Crucially, as we discuss in detail in our estimation Appendix C, the process of constructing our cross-sectional data proxy for microeconomic uncertainty involves time aggregation from quarterly to annual frequency, an unavoidable temporal mismatch of the measurement of inputs and outputs within the year, as well as measurement error of productivity in the underlying Census of Manufactures sample. In Appendix C we demonstrate within the model that each of these measurement steps leads to a reduction in the variability of the uncertainty proxy relative to its mean level, with the temporal mismatch between input and output measurement, as well as measurement error itself, accounting for the bulk of the shift. The large increase in microeconomic uncertainty $\sigma_t^Z$ which we estimate upon impact of an uncertainty shock is critical for matching the behavior of measured productivity shock dispersion in the data. As Table 6 demonstrates, the estimated model captures the overall time series properties of measured uncertainty in our data quite closely.

5 Quantitative Analysis

In what follows we explore the quantitative implications of our model. We begin by discussing the unconditional second moments generated by the model. We then continue by specifically studying the effects of an uncertainty shock.

5.1 Business Cycle Statistics

Table 7 illustrates that the model generates second-moment statistics that resemble their empirical counterparts in U.S. data. We simulate the model over 5000 quarters using the histogram or nonstochastic simulation approach following Young (2010). We then compute the standard set of business cycle statistics, after discarding an initial 500 quarters. As in the data, investment and hours commove with output. Output and consumption commove, although not as much as in the data. Investment is more volatile than output, while consumption is less volatile. Given the high assumed Frisch elasticity of labor supply, the model also generates a realistic volatility of hours relative to output. See Rogerson 1988, Hansen 1985, or Benhabib, et al. 1991 for discussion of underlying mechanisms which can generate more elastic labor supply in this class of models. Overall, we conclude that the business cycle implications of our model are consistent with the common findings in the literature.
5.2 The Effects of an Uncertainty Shock

As has been known since at least Scarf (1959), non-convex adjustment costs lead to \( S_s \) investment and hiring policy rules. Firms do not hire and invest until productivity reaches an upper threshold (the \( S \) in \( S_s \)) or fire and disinvest until productivity hits a lower threshold (the \( s \) in \( S_s \)). This is shown for labor in Figure 5, which plots the distribution of firms by their productivity/labor, \( \frac{A_x}{n_{-1}} \), ratios after the micro and macro shocks have been drawn but before firms have adjusted. On the right is the firm-level hiring threshold (right solid line) and on the left the firing threshold (left solid line), in the case of low uncertainty. Firms to the right of the hiring line will hire, firms to the left of the firing line will fire, and those in the middle will be inactive for the period.

An increase in uncertainty increases the returns to inaction, shown by the increased hiring threshold (right dotted line) and reduced firing threshold (left dotted line). When uncertainty is high firms become more cautious as labor adjustment costs make it expensive to make a hiring or firing mistake. Hence, the hiring and firing thresholds move out, increasing the range of inaction. This leads to a fall in net hiring, since the mass of firms is right-shifted due to labor attrition. A similar phenomenon occurs with capital, whereby increases in uncertainty reduce the amount of net investment.

5.2.1 Modeling a Pure Uncertainty Shock

To analyze the aggregate impact of uncertainty we independently simulate 2500 economies, each of 100-quarter length. The first 50 periods are simulated unconditionally, so all exogenous processes evolve normally. Then for each economy after 50 quarters we insert an uncertainty shock by imposing a high uncertainty state. From the shock period onwards each economy evolves normally. To calculate the impulse response function to an uncertainty shock for any macro variable, we first compute the average of the aggregate variable in each period \( t \) across simulated economies. The effect of an uncertainty shock is then simply given by the percentage deviation of the average in period \( t \) from its value in the pre-shock period.

Figure 6 depicts the impact of an uncertainty shock on output. For graphical purposes period “0” in the figure corresponds to the pre-shock period in the above discussion, i.e. quarter 50. Figure 6 displays a drop in output of just over 2.5% within one quarter. This significant fall is one of the key results of the paper as it shows that uncertainty shocks can be a quantitatively important contributor to business cycles within a general equilibrium framework. A quick recovery follows the initial decline, and output returns back to normal levels within one year. We note that output then declines again moderately from quarters 6 onwards. We defer the discussion for the intuition behind this result until Section 5.2.4.
These dynamics in output arise from the dynamics in three channels: labor, capital, and the misallocation of factors of production. These are depicted in Figure 7. First, in the top-left panel we plot the time path of hours worked. When uncertainty increases most firms pause hiring, and hours worked begin to drop because workers are continuing to attrit from firms without being replaced. In the model this rate of exogenous attrition is assumed to be constant over the cycle. This is consistent with Shimer (2005) and Hall (2005), which show that around three quarters of the movements in the volatility of unemployment are due to job-finding rates and not to the cyclicality of the destruction rate. Similarly, in the top-right panel we plot the time path of investment, which drops rapidly due to the increase in uncertainty. Since investment falls but capital continues to depreciate, there is also a drop in the capital stock.

Misallocation of factor inputs - using the terminology of Hsieh and Klenow (2009) - increases in the economy in response to an uncertainty shock. In normal times, unproductive firms contract and productive firms expand, helping to maintain high levels of aggregate productivity. But when uncertainty is high, firms reduce expansion and contraction, shutting off much of this productivity-enhancing reallocation. In the bottom-left panel, we plot the path of the dispersion of the marginal product of labor after an uncertainty shock. More precisely, the bottom-left panel plots the impulse response of the cross-sectional standard deviation of $\log (\frac{z}{n})$. In the wake of an uncertainty shock, labor misallocation endogenously worsens, improving only slowly. In the longer run, labor, investment, misallocation, and output all start to recover to their steady state as the uncertainty shock is temporary. As uncertainty falls back, firms start to hire and invest again to address their pent-up demand for labor and capital. In Figure B3 in the appendix we depict alternative measures of misallocation. As Figure B3 shows, all of these alternative measures point to the same result of increased misallocation following an uncertainty shock.

In the lower-right panel of Figure 7 we plot the time profile of consumption. When the uncertainty shock occurs, consumption jumps up in the first quarter before subsequently falling back below the mean of the ergodic distribution for several quarters. The logic behind this initial increase in consumption is as follows. In the impact period of the uncertainty shock, i.e., period 1 in Figure 7, the households understand that the degree of misallocation has increased in the economy as the bottom-left panel demonstrates. Increased misallocation acts as a negative first-moment shock to aggregate productivity and thus lowers the expected return on savings, making immediate consumption more attractive and thus leading to its first-period increase. Furthermore, this jump in consumption is feasible since in the impact period of the uncertainty shock the freeze in both investment and hiring reduces the resources spent on capital and adjustment costs, thus freeing up resources. After this initial jump, starting in period 2 in Figure 7 the capital stock is now below its ergodic
distribution, where we note that in the impact period of the uncertainty shock the pre-shock aggregate capital level was fixed. This fact, together with hours worked being below their ergodic distribution and the degree of misallocation being above its ergodic distribution, limits the overall resources in the economy and thus limits consumption. In addition, the economy begins its recovery in period 2 in Figure 7, which is manifested in investment and hiring beginning to increase relative to the values exhibited during the impact period of the uncertainty shock. This recovery requires resources to be spent on capital and adjustment costs, further reducing the available resources for consumption. Interestingly, we note that Khan and Thomas (2013) find in their model of credit constraints that while output, labor, and investment fall in response to a credit tightening shock, in fact consumption, as in our model, also initially rises due to similar general equilibrium effects.

Clearly, this rise in consumption at the start of recessions is an unattractive feature of a pure uncertainty shock model of business cycles. Several options exist, however, to try and address this. One is to allow consumers to save in other technologies besides capital, for example, in foreign assets. This is the approach Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011) take in modelling risk shocks in small open economies. In an open economy model a domestic uncertainty shock induces agents to increase their savings abroad (capital flight). In our closed model this is not possible, but extending the model to allow a foreign sector would make this feasible although computationally more intensive. Another option would be to use utility functions such as those in Greenwood, Hercowitz, and Huffman (1998). Due to the complementarity between consumption and hours in such preference structures, they should reduce the overshoot in consumption. We do not explore these options for the following two reasons. Having another investment vehicle such as foreign bond would add an additional state variable to the problem, and switching the preference structure would require us to forecast wages separately from marginal utility. Both changes would increase the computational burden considerably. Another option would be to model precautionary behavior from households in the wake of an uncertainty shock, as Basu and Bundick (2016) do in a New Keynesian environment with demand-determined output. Such behavior would allow for natural investment, consumption, and output co-movement, but expanding the aggregate structure of the model to account for nominal rigidities is beyond the scope of the current paper.

5.2.2 First-Moment and Second-Moment Shocks

Our quantitative results so far reveal that uncertainty shocks can contribute importantly to recessions, but there are at least two unattractive implications of modeling an uncertainty shock in isolation. First, the empirical evidence in Section 2 suggests that recessions are periods of both first- and second-moment shocks, at least to the extent that average
outcomes decrease and variances of innovations to establishment-level TFP increase during recessions. And second, in our model uncertainty shocks are associated with an increase in consumption on impact and a reduction in disinvestment and firing (see Figures 5 and 7).

Thus, to generate an empirically more realistic simulation we consider the combination of an uncertainty shock and a −2% exogenous first moment shock. See Appendix B for a discussion of the specific numerical experiment considered here.

As Figure 8 suggests, this additional shock magnifies the drop in output, investment, and hours. The addition of the first-moment shock also leads to a fall in consumption on impact. Given the business cycle comovement we observe empirically, we conclude that simultaneous first- and second-moment shocks in the model can generate dynamics that resemble recent US recessions.

5.2.3 Robustness

In this section we discuss the robustness of our finding to different parameterizations. We first consider the robustness of our results with respect to the estimated parameters governing the uncertainty process. Since the estimated values in Section 4.2 pointed to (i) a significant jump in both micro and macro uncertainty, (ii) a significant persistence of the uncertainty process, and (iii) a moderately high frequency of high uncertainty, we consider experiments where we reduce the values of each of these parameters. For consistency unless otherwise noted every robustness experiment considers a reduction of 25% in the parameter value. Thus, within this set we consider (i) a reduction of the macro volatility jump, $\frac{\sigma_A^2}{\sigma_L^2}$, from 1.6 to 1.2, (ii) a reduction of the micro volatility jump, $\frac{\sigma_H^2}{\sigma_L^2}$, from 4.13 to 3.10, (iii) a reduction in the likelihood of transition from low uncertainty to high uncertainty, $\pi_{L,H}^\tau$, from 0.026 to 0.02, (iv) a reduction in the uncertainty persistence, $\pi_{H,H}^\tau$, from 0.94 to 0.71. Finally we also consider two additional experiments where (i) we lower the micro volatility baseline value, $\sigma_L^2$, from 0.051 to 0.038 and (ii) where we lower macro volatility baseline value, $\sigma_A^2$, from 0.0067 to 0.0050. Figure 9 plots the effects of each of these variations on output, investment, labor, and consumption. As Figure 9 suggests the results are overall robust to these changes, preserving the dynamics reported in Figures 6-8. The one exception is the reduction in the persistence of the uncertainty shock to 0.7 from the estimated value of 0.94. At lower levels of persistence the impact is short-lived. This highlights how the dynamics of impact of uncertainty shocks are sensitive to the persistence of the underlying shock and is another motivation for our SMM estimation of the parameters of the uncertainty process. We note that the estimated value of 0.94 for the autocorrelation of uncertainty may seem high. However, this potentially accounts for the endogenous amplification of uncertainty from slower growth proposed by a range of other papers.

In addition to systematically plotting robustness checks to changes in the value of each
of our six estimated uncertainty process parameters, Figure 9 also plots the results from one additional experiment in which we reduce the value of all capital and labor adjustment cost parameters by 25% simultaneously. The effect of an uncertainty shock changes little relative to our baseline, a result which reflects the fact, well known within the wide literature on investment and adjustment costs, that the size of inaction regions are concave functions of adjustment costs. For example, see the analytical results in Dixit (1995) and Abel and Eberly (1996) showing that the size of the inaction region for investment in their models expands to the third or fourth order in adjustment costs around zero.

5.2.4 Decomposing the Impact of Uncertainty

The next set of robustness results studies how the effects of uncertainty shocks differ across general equilibrium (GE) and partial equilibrium (PE) frameworks. To address this question we plot in Figure 10 the impact of an uncertainty shock in three different economies. The black line (× symbols) depicts again the effects of an uncertainty shock in our GE model economy, the red line (+ symbols) depicts the same response but with PE only (all prices and wages are held constant and the consumer side of the economy ignored), while the blue line (o symbols) depicts the effects of an uncertainty shock in a PE economy with no adjustment costs at all. Note that in the bottom-right panel in the PE economies consumption is not defined because there is no notion of an aggregate household. We therefore impose a zero response path for consumption in those cases.

When there are no adjustment costs of any type in a PE economy, output actually increases following an uncertainty shock. The reason for this result is related to the Oi (1961), Hartman (1972), and Abel (1983) effect, whereby a higher variance of productivity increases investment, hiring, and output because the optimal capital and labor choices are convex in productivity.

By contrast, the addition of adjustment costs to the PE setup dramatically changes the effect of an uncertainty shock. Now, on impact there is a fall in aggregate output. The reason is that the increase in uncertainty moves firms’ labor and capital Ss bands out, temporarily pausing hiring and investment. If all firms pause hiring and investment, aggregate labor and capital drop due to labor attrition and capital depreciation. But this pause is short-lived, as once uncertainty drops back firms start to hire and invest again. So in the medium-run the Oi-Hartman-Abel effect dominates and output rises above its long-run trend.

While these forces are also present in the baseline GE adjustment cost economy, the curvature in the utility function, i.e. the endogenous movement in the interest rate, moderates the rebound and overshoot. The overshoot in the PE economy requires big movements in investment and labor, which are feasible since consumption is not taken into account in
the PE framework. However, in the GE framework, the curvature in utility slows down the rebound of the GE economy, generating a smoother and more persistent output cycle. Intriguingly, in the first period, however, GE has very little impact on output relative to the PE economy with adjustment cost. This is because the $S_s$ bands have moved so far out that there is a reduced density of firms near the hiring or investment thresholds to respond to prices. Hence, the short-run robustness of the impact of uncertainty shocks to GE suggested by Bloom (2009) seems to be present, while the medium-run sensitivity to GE highlighted by Thomas (2002) and Khan and Thomas (2003, 2008) is also present.

We are now in position to discuss the reason for the sluggish behavior of output in the medium-term after an uncertainty shock in Figure 6. First, on impact in periods 1-2 the “real options” effect due to the uncertainty shock dominates, leading to a hiring/investment freeze, misallocation, and thus to a significant drop in output. Later on in periods 3-5, the economy exhibits a “rebound” as the high micro volatility is realized and some firms draw significantly higher productivity shocks than before. Firms start to readjust and the Oi-Hartman-Abel effect leads to a recovery. During the third stage starting in periods 6-8 output declines again.

Two factors play a role in the second fall in output. First, the level of misallocation remains high, which acts as a drag on output and is a large contributor to the slowdown of the recovery. The bottom left panel of Figure 7 reveals that the cross-sectional dispersion of the marginal product of labor - a measure of labor misallocation - remains almost 10% higher in period 6 and declines only slowly as firms begin to adjust their inputs again in the wake of an uncertainty shock. Underlying misalignment of inputs and productivity at the micro level prevents effective use of the capital and labor stock of the aggregate economy, and meaningful input adjustment costs prevent such misallocation from resolving itself quickly over this period.

The second factor contributing to the second more gradual decline in output is a declining path for investment starting around period 6. By this stage, the real options effect has subsided in large part, and the economy has a low but growing consumption path. This result is a declining path of interest rates over which it is optimal to have a declining investment path. These investment dynamics of the economy in the third stage of the response to an uncertainty shock resemble those that show up in a basic neoclassical growth model as in Brock and Mirman (1972). Figure D1 in Appendix D plots the response of such an economy to a capital destruction shock. As the figure shows, consumption converges from below at a declining rate, and investment declines over the path.
5.2.5 Laws of Motion Robustness

Our final set of robustness checks studies the impact of variations in the labor and capital depreciation rates. In these experiments we vary one by one the capital depreciation rate and the labor depreciation rate by one quarter each. The results are depicted in Figure 11. As the figure suggests, unsurprisingly reduction of the labor depreciation rate attenuates the fall in hours and thus in output but preserves the overall dynamics found in the benchmark calibration. Changes in the capital depreciation rate do not change the fall on impact, which is driven by the behavior in labor. Later on, a lower capital depreciation rate accentuates the recovery from an uncertainty shock as it induces a strong rebound in investment leading to an increase in capital and thus in labor and output. However we note that the more empirically relevant exercise is to increase the capital depreciation rate since investment is increasingly shifting towards intangible areas, which have much higher depreciation rates. For example, the BEA’s migration of R&D from the satellite accounts to the fixed assets tables is based on depreciation rates of around 20% or higher (Li 2012). Thus, while the reduction in annual depreciation does reduce the impact of an uncertainty shock, the empirically relevant depreciation rates in the economy is likely much higher.

6 Policy in the Presence of Uncertainty

In this section, we analyze the effects of stimulative policies in the presence of uncertainty shocks. It is important to emphasize that any such policy is not optimal within the context of our model, as the competitive equilibrium is Pareto optimal. Rather, we see our policy experiments as a means of documenting and quantifying the effects of such policies in times of heightened uncertainty. It is also worth noting that this ignores the direct impact of policy on uncertainty as studied by papers such as Baker, Bloom, and Davis (2016), Baker and Bloom (2011), and Hassan, et al. (2017).

The policy experiment we consider is a policy that attempts to temporarily stimulate hiring by reducing the effective wage paid by firms. More specifically, the policy consists of an unanticipated 1% wage bill subsidy paid for one quarter and financed through a lump-sum tax on households. We simulate this policy impulse once during an uncertainty shock and also in an economy that is not hit by an uncertainty shock. By comparing the marginal effect in those two cases, we attempt to identify the effect of uncertainty on policy effectiveness.

Figure 12 depicts this experiment as it shows the net impact of the policy. That is, we first solve for the effects of the policy on output when it does not coincide with an uncertainty shock. Subtracting from this the behavior of output when there is no uncertainty shock and no subsidy yields the net impact of the policy in the absence of an uncertainty shock.
We then solve for the policy’s effect when it \emph{does} coincide with an uncertainty shock. Similarly, subtracting from this latter experiment the behavior of output when there is an uncertainty shock and no subsidy, the behavior depicted in Figure 6, yields the net impact of the policy in the presence of an uncertainty shock. As Figure 12 shows, the presence of uncertainty reduces the effects of the wage policy by over two thirds on impact. The reason is that as soon as uncertainty rises, the $S$s thresholds jump out, so many firms are far away from their hiring and investment thresholds, making them less responsive to any policy stimulus. Our result here echo findings from the lumpy investment literature on the procyclicality of the investment response to productivity shocks (e.g. Bachmann, Caballero, and Engel 2013). In particular, we show that uncertainty or second-moment shocks in addition to first-moment shocks can also generate movement in the responsiveness of the economy to shocks. Interestingly, in the context of a New Keynesian economy with a distinct structure and policy experiment Vavra (2014) also finds that second-moment shocks reduce the responsiveness of the economy to policy.

Overall, our results highlight how uncertainty shocks lead to time-varying policy effectiveness. At the instant an uncertainty shock hits, policy is not as effective relative to normal times. Hence, uncertainty shocks not only impact the economy directly but also indirectly change the response of the economy to any potential reactive stabilization policy.

7 Conclusions

Uncertainty has received substantial attention as a potential factor in business cycles. The first part of this paper uses Census microdata to show that measured uncertainty is indeed strongly countercyclical. This is true both at the aggregate and the industry-level: slower industry growth is associated with higher industry uncertainty.

The second part of the paper then builds a DSGE model with heterogeneous firms, time-varying uncertainty, and adjustment costs to quantify the impact of second-moment shocks. We find that uncertainty shocks typically lead to drops of about 2.5% in GDP, with a sharp drop, quick recovery, then continued sluggishness in output. This suggests that uncertainty could play an important role in driving business cycles, either as an impulse or amplification mechanism. We also find that because uncertainty makes firms cautious, the response of the economy to stimulative policy substantially declines. Finally, both our empirical and simulation results suggest recessions are best modelled as a combination of a negative first-moment and a positive second-moment shock.
References


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Table 1: Uncertainty is Higher During Recessions

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Dependent Variable:</th>
<th>(1) S.D. of log(TFP) shock</th>
<th>(2) Skewness of log(TFP) shock</th>
<th>(3) Kurtosis of log(TFP) shock</th>
<th>(4) IQR of log(TFP) shock</th>
<th>(5) IQR of output growth</th>
<th>(6) IQR of sales growth</th>
<th>(7) IQR of stock returns</th>
<th>(8) IQR of industrial prod. Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td></td>
<td>0.064***</td>
<td>-0.248</td>
<td>-1.334</td>
<td>0.061***</td>
<td>0.077***</td>
<td>0.032***</td>
<td>0.025***</td>
<td>0.044***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.191)</td>
<td>(1.994)</td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.007)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Mean of Dep. Var.</td>
<td></td>
<td>0.503</td>
<td>-1.525</td>
<td>20.293</td>
<td>0.395</td>
<td>0.196</td>
<td>0.186</td>
<td>0.104</td>
<td>0.101</td>
</tr>
<tr>
<td>Cor. GDP growth</td>
<td></td>
<td>-0.450***</td>
<td>0.143</td>
<td>0.044</td>
<td>-0.444***</td>
<td>-0.566***</td>
<td>-0.275***</td>
<td>-0.297***</td>
<td>-0.335***</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td>Annual</td>
<td>Annual</td>
<td>Annual</td>
<td>Annual</td>
<td>Annual</td>
<td>Quarterly</td>
<td>Monthly</td>
<td>Monthly</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>191</td>
<td>609</td>
<td>455</td>
</tr>
</tbody>
</table>

Notes: Each column reports a time-series OLS regression point estimate (and standard error below in parentheses) of a measure of uncertainty on a recession indicator. The recession indicator is the share of quarters in that year in a recession in columns (1) to (5), whether that quarter was in a recession in column (6), and whether the month was in recession in columns (7) and (8). Recessions are defined using the NBER data. In the bottom panel we report the mean of the dependent variable and its correlation with real GDP growth. In columns (1) to (5) the sample is the population of manufacturing establishments with 25 years or more of observations in the ASM or CM survey between 1972 and 2009, which contains data on 15,673 establishments across 39 years of data (one more year than the 38 years of regression data since we need lagged TFP to generate a TFP shock measure). We include plants with 25+ years to reduce concerns over changing samples. In column (1) the dependent variable is the cross-sectional standard deviation (S.D.) of the establishment-level ‘shock’ to Total Factor Productivity (TFP). This ‘shock’ is calculated as the residual from the regression of log(TFP) at year t+1 on its lagged value (year t), a full set of year dummies and establishment fixed effects. In column (2) we use the cross-sectional coefficient of skewness of the TFP ‘shock’, in column (3) the cross-sectional coefficient of kurtosis and in column (4) the cross-sectional interquartile range of this TFP ‘shock’ as an outlier robust measure. In column (5) the dependent variable is the interquartile range of plants’ sales growth. In column (6) the dependent variable is the interquartile range of firms’ sales growth by quarter for all public firms with 25 years (100 quarters) or more in Compustat between 1962 and 2010. In column (7) the dependent variable is the within firm-quarter interquartile range of firms’ monthly stock returns for all public firms with 25 years (300 months) or more in CRSP between 1960 and 2010. Finally, in column (8) the dependent variable is the interquartile range of industrial production growth by month for manufacturing industries from the Federal Reserve Board’s monthly industrial production database. All regressions include a time trend and for columns (1) to (5) Census year dummies (for Census year and for 3 lags). Robust standard errors are applied in all columns to control for any potential serial correlation. *** denotes 1% significance, ** 5% significance and * 10% significance. Results are also robust to using Newey-West corrections for the standard errors. Data available on-line at http://www.stanford.edu/~nbloom/RUBC.zip.
Table 2: Uncertainty is Also Robustly Higher at the Industry Level during Industry ‘Recessions’

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification:</td>
<td>Baseline</td>
<td>Median industry output growth</td>
<td>IQR of establishment TFP shocks within each industry-year cell</td>
<td>Median industry output growth</td>
<td>IQR of industry establishment size</td>
<td>Median industry establishment size</td>
<td>IQR of industry establishment size</td>
<td>Median industry capital/labor ratio</td>
<td>IQR of industry capital/labor ratio</td>
</tr>
<tr>
<td>Industry Output Growth</td>
<td>-0.132***</td>
<td>-0.142***</td>
<td>-0.176***</td>
<td>-0.119***</td>
<td>-0.116***</td>
<td>-0.111***</td>
<td>-0.111***</td>
<td>-0.191***</td>
<td>-0.133***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.047)</td>
<td>(0.024)</td>
<td>(0.022)</td>
<td>(0.034)</td>
<td>(0.030)</td>
<td>(0.041)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Interaction of industry output growth with the variable in specification row</td>
<td>0.822</td>
<td>0.882</td>
<td>-0.032</td>
<td>-0.033</td>
<td>-0.197</td>
<td>0.123</td>
<td>0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.630)</td>
<td>(0.996)</td>
<td>(0.038)</td>
<td>(0.026)</td>
<td>(0.292)</td>
<td>(0.330)</td>
<td>(0.084)</td>
<td>(0.122)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying sample</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
<td>446,051</td>
</tr>
</tbody>
</table>

Notes: Each column reports the results from an industry-by-year OLS panel regression, including a full set of industry and year fixed effects. The dependent variable in every column is the interquartile range (IQR) of establishment-level TFP ‘shocks’ within each SIC 4-digit industry-year cell. The regression sample is the 16,451 industry-year cells of the population of manufacturing establishments with 25 years or more of observations in the ASM or CM survey between 1972 and 2009 (which contains 446,051 underlying establishment years of data). These industry-year cells are weighted in the regression by the number of establishment observations within that cell, with the mean and median number of establishments per industry-year cell 27.1 and 17 respectively. The TFP ‘shock’ is calculated as the residual from the regression of log(TFP) at year t+1 on its lagged value (year t), a full set of year dummies and establishment fixed effects. In column (1) the explanatory variable is the median of the establishment-level output growth in that industry-year. In columns (2) to (9) a second variable is also included which is an interaction of that explanatory variable with an industry-level characteristic. In columns (2) and (3) this is the median and IQR of industry-level output growth, in columns (4) and (5) this is the median and IQR of industry-level establishment size in employees, in columns (6) and (7) this is the median and IQR of industry-level capital/labor ratios, in column (8) this is the IQR of industry-level TFP levels (note the mean is zero by construction), while finally in column in (9) this interaction is the dispersion of industry-level concentration measured using the Ellison-Glaeser dispersion index. Standard errors clustered by industry are reported in brackets below every point estimate. *** denotes 1% significance, ** 5% significance and * 10% significance.
### Table 3: Cross-Sectional Establishment Uncertainty Measures are Correlated with Firm and Industry Time Series Uncertainty Measures

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of establishment absolute (TFP shocks) within firm year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean of establishment absolute (TFP shocks) within industry year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample</td>
<td>Establishments (in manufacturing) with a parent firm in Compustat</td>
<td></td>
<td></td>
<td></td>
<td>Manufacturing industries</td>
</tr>
<tr>
<td>Regression panel dimension</td>
<td>Firm by Year</td>
<td>Firm by Year</td>
<td>Firm by Year</td>
<td>Industry by Year</td>
<td></td>
</tr>
<tr>
<td>S.D. of parent daily stock returns within year</td>
<td>0.317***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.D. of parent monthly stock returns within year</td>
<td>0.275***</td>
<td>0.381***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.083)</td>
<td>(0.118)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.D. of parent daily stock returns within year, leverage adjusted</td>
<td></td>
<td></td>
<td></td>
<td>0.134***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.029)</td>
<td></td>
</tr>
<tr>
<td>S.D. of parent quarterly sales growth within year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.D. of monthly industrial production within year</td>
<td></td>
<td></td>
<td></td>
<td>0.330***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.060)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fixed effects and clustering</th>
<th>Firm</th>
<th>Firm</th>
<th>Firm</th>
<th>Firm</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms/Industries</td>
<td>1,838</td>
<td>1,838</td>
<td>1,838</td>
<td>1,838</td>
<td>466</td>
</tr>
<tr>
<td>Observations</td>
<td>25,302</td>
<td>25,302</td>
<td>25,302</td>
<td>25,302</td>
<td>446,051</td>
</tr>
<tr>
<td>Underlying observations</td>
<td>172,074</td>
<td>172,074</td>
<td>172,074</td>
<td>172,074</td>
<td>16,406</td>
</tr>
</tbody>
</table>

**Notes:** The dependent variable is the mean of the absolute size of the TFP shock at the firm-year level (columns (1) to (4)) and industry-year level (column (5)). This TFP shock is calculated as the residual from the regression of log(TFP) at year t+1 on its lagged value (year t), a full set of year dummies and establishment fixed effects, with the absolute size generating by turning all negative values positive. The regression sample in columns (1) to (4) are the 25,302 firm-year cells of the population of manufacturing establishments with 25 years or more of observations in the ASM or CM survey between 1972 and 2009 which are owned by Compustat (publicly listed) firms. This covers 172,074 underlying establishment years of data. The regression sample in column (5) is the 16,406 industry-year cells of the population of manufacturing establishments with 25 years or more of observations in the ASM or CM survey between 1972 and 2009. The explanatory variables in columns (1) to (3) are the annual standard deviation of the parent firm’s stock returns, which are calculated using the 260 daily values in columns (1) and (3) and the 12 monthly values in column (2). For comparability of monthly and daily values, the coefficients and S.E for the daily returns columns (1) and (3) are divided by sqrt(21). The daily stock returns in column (2) are normalized by the (equity/(debt+equity)) ratio to control for leverage effects. In column (4) the explanatory variable is the standard deviation of the parent firm’s quarterly sales growth. Finally, in column (5) the explanatory variable is the standard deviation of the industry’s monthly industrial production data from the Federal Reserve Board. All columns have a full set of year fixed effects with columns (1) to (4) also having firm fixed effects while column (5) has industry fixed effects. Standard errors clustered by firm/industry are reported in brackets below every point estimate. *** denotes 1% significance, ** 5% significance and * 10% significance.
### Table 4: Calibrated Model Parameters

<table>
<thead>
<tr>
<th>Preferences and Technology</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.95 (1/4)</td>
<td>Annual discount factor of 95%</td>
</tr>
<tr>
<td>η</td>
<td>1</td>
<td>Unit elasticity of intertemporal substitution (Khan and Thomas 2008)</td>
</tr>
<tr>
<td>θ</td>
<td>2</td>
<td>Leisure preference, households spend 1/3 of time working</td>
</tr>
<tr>
<td>χ</td>
<td>1</td>
<td>Infinite Frisch elasticity of labor supply (Khan and Thomas 2008)</td>
</tr>
<tr>
<td>α</td>
<td>0.25</td>
<td>CRS production, isoelastic demand with 33% markup</td>
</tr>
<tr>
<td>ν</td>
<td>0.5</td>
<td>CRS labor share of 2/3, capital share of 1/3</td>
</tr>
<tr>
<td>ρ\textsuperscript{A}</td>
<td>0.95</td>
<td>Quarterly persistence of aggregate productivity (Khan and Thomas 2008)</td>
</tr>
<tr>
<td>ρ\textsuperscript{Z}</td>
<td>0.95</td>
<td>Quarterly persistence of idiosyncratic productivity (Khan and Thomas 2008)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adjustment Costs</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ\textsubscript{k}</td>
<td>2.6%</td>
<td>Annual depreciation of capital stock of 10%</td>
</tr>
<tr>
<td>δ\textsubscript{n}</td>
<td>8.8%</td>
<td>Annual labor destruction rate of 35% (Shimer 2005)</td>
</tr>
<tr>
<td>F\textsuperscript{K}</td>
<td>0</td>
<td>Fixed cost of changing capital stock (Bloom 2009)</td>
</tr>
<tr>
<td>S</td>
<td>33.9%</td>
<td>Resale loss of capital in % (Bloom 2009)</td>
</tr>
<tr>
<td>F\textsuperscript{L}</td>
<td>2.1%</td>
<td>Fixed cost of changing hours in % of annual sales (Bloom 2009)</td>
</tr>
<tr>
<td>H</td>
<td>1.8%</td>
<td>Per worker hiring/firing cost in % of annual wage bill (Bloom 2009)</td>
</tr>
</tbody>
</table>

**Notes:** The model parameters relating to preferences, technology, and adjustment costs are calibrated as referenced above.
### Table 5: Estimated Uncertainty Parameters

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^A_L$</td>
<td>0.67</td>
<td>(0.098)</td>
</tr>
<tr>
<td>$\sigma^A_H/\sigma^A_L$</td>
<td>1.6</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\sigma^Z_L$</td>
<td>5.1</td>
<td>(0.807)</td>
</tr>
<tr>
<td>$\sigma^Z_H/\sigma^Z_L$</td>
<td>4.1</td>
<td>(0.043)</td>
</tr>
<tr>
<td>$\pi^\sigma_{L,H}$</td>
<td>2.6</td>
<td>(0.485)</td>
</tr>
<tr>
<td>$\pi^\sigma_{H,H}$</td>
<td>94.3</td>
<td>(16.38)</td>
</tr>
</tbody>
</table>

Quarterly standard deviation of macro productivity shocks, %
Macro volatility increase in high uncertainty state
Quarterly standard deviation of micro productivity shocks, %
Micro volatility increase in high uncertainty state
Quarterly transition probability from low to high uncertainty, %
Quarterly probability of remaining in high uncertainty, %

**Notes:** The uncertainty process parameters are structurally estimated through a SMM procedure (see the main text and Online Appendix C). The estimation process targets the time series moments of the cross-sectional interquartile range of the establishment-level shock to estimated productivity in the Census of Manufactures and Annual Survey of Manufactures manufacturing sample, along with the time series moments of estimated heteroskedasticity of the US aggregate Solow residual based on a GARCH(1,1) model. Both sets of target moments from the data are computed from 1972-2010.

### Table 6: Uncertainty Process Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macro Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>3.36</td>
<td>3.58</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.76</td>
<td>0.59</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.83</td>
<td>1.18</td>
</tr>
<tr>
<td>Serial Correlation</td>
<td>0.88</td>
<td>0.83</td>
</tr>
<tr>
<td><strong>Micro Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>39.28</td>
<td>38.44</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.89</td>
<td>4.55</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.16</td>
<td>0.81</td>
</tr>
<tr>
<td>Serial Correlation</td>
<td>0.75</td>
<td>0.65</td>
</tr>
</tbody>
</table>

**Notes:** The micro data moments are calculated from the US Census of Manufactures and Annual Survey of Manufactures sample using annual data from 1972-2010. Micro data moments are computed from the cross-sectional interquartile range of the estimated shock to establishment-level productivity, in percentages. The model micro moments are computed in the same fashion as the data moments, after correcting for measurement error in the data establishment-level regressions and aggregating to annual frequency. The macro data moments refer to the estimated heteroskedasticity from 1972-2010 implied by a GARCH(1,1) model of the annualized quarterly change in the aggregate US Solow residual, with quarterly data downloaded from John Fernald's website. The model macro moments are computed from an analogous GARCH(1,1) estimation on simulated aggregate data. All model results are based on a simulation of 1000 firms for 5000 quarters, discarding the first 500 periods.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma(x)$</td>
<td>$\sigma(y)$</td>
<td>$\rho(x,y)$</td>
<td>$\sigma(x)$</td>
</tr>
<tr>
<td>Output</td>
<td>1.6</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Investment</td>
<td>7.0</td>
<td>4.5</td>
<td>0.9</td>
<td>11.9</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.3</td>
<td>0.8</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Hours</td>
<td>2.0</td>
<td>1.3</td>
<td>0.9</td>
<td>2.4</td>
</tr>
</tbody>
</table>

**Notes:** The first panel contains business cycle statistics for quarterly US data covering 1972Q1-2010Q4. All business cycle data is current as of July 14, 2014. Output is real gross domestic product (FRED GDPC1), investment is real gross private domestic investment (GPDIC1), consumption is real personal consumption expenditures (PCECC96), and hours is total nonfarm business sector hours (HOANBS). The second panel contains business cycle statistics from unconditional simulation of the estimated model, computed from a 5000-quarter simulation with the first 500 periods discarded. All series are HP-filtered with smoothing parameter 1600, in logs expressed as percentages.
Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures using a balanced panel of 15,752 establishments active in 2005-06 and 2008-09. TFP Shocks are defined as residuals from a plant-level log(TFP) AR(1) regression that also includes plant and year fixed effects. Moments of the distribution for non-recession (recession) years are: mean 0 (-0.166), variance 0.198 (0.349), coefficient of skewness -1.060 (-1.340) and kurtosis 15.01 (11.96). The year 2007 is omitted because according to the NBER the recession began in 12/2007, so 2007 is not a clean “before” or “during” recession year.

Figure 1: The variance of establishment-level TFP shocks increased by 76% in the Great Recession
Figure 2: The variance of establishment-level sales growth rates increased by 152% in the Great Recession.

Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures using a balanced panel of 15,752 establishments active in 2005-06 and 2008-09. Moments of the distribution for non-recession (recession) years are: mean 0.026 (-0.191), variance 0.052 (0.131), coefficient of skewness 0.164 (-0.330) and kurtosis 13.07 (7.66). The year 2007 is omitted because according to the NBER the recession began in 12/2007, so 2007 is not a clean “before” or “during” recession year.
Figure 3: TFP ‘shocks’ are more dispersed in recessions

Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures establishments, using establishments with 25+ years to address sample selection. Grey shaded columns are the share of quarters in recession within a year.
Figure 4: Robustness test: different measures of TFP ‘shocks’ are all more dispersed in recessions

Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures establishments, using establishments with 25+ years to address sample selection. Grey shaded columns are share of quarters in recession within a year. The four lines are: Baseline: Interquartile Range of plant TFP ‘shocks’ (as in Figure 3). Add polynomials in TFP: includes the first, second and third lags of log TFP, and their 5 degree polynomials in the AR regression which is used to recover TFP shocks. Add investment: includes all the controls from the previous specification plus the first, second and third lags of investment rate, and their 5 degree polynomials. Add emp, sales and materials: includes all the controls from the previous specification plus the second and third lags of log employment, log sales, and log materials, as well as their 5 degree polynomials.
Figure 5: The impact of an increase in uncertainty on the hiring and firing thresholds

Notes: The figure plots the simulated cross-sectional marginal distribution of micro-level labor inputs after productivity shock realizations and before labor adjustment. The distribution plots a representative period with average aggregate productivity and low uncertainty levels. The vertical hiring and firing thresholds are computed based on firm policy functions with average micro-level productivity realizations, taking as given the aggregate state of the economy with low uncertainty (solid lines) and a high uncertainty counterfactual (dotted lines).
Notes: Based on independent simulations of 2500 economies of 100-quarter length. We impose an uncertainty shock in the quarter labelled 1, allowing normal evolution of the economy afterwards. We plot the percent deviation of cross-economy average output from its value in quarter 0.
Figure 7: Labor and investment drop and rebound, misallocation rises, and consumption overshoots then falls.

Notes: Based on independent simulations of 2500 economies of 100-quarter length. We impose an uncertainty shock in the quarter labelled 1, allowing normal evolution of the economy afterwards. Clockwise from the top left, we plot the percent deviations of cross-economy average labor, investment, consumption, and the dispersion of the marginal product of labor from their values in quarter 0.
Figure 8: Adding a -2% first-moment shock increases the output fall and eliminates a consumption overshoot

Notes: Based on independent simulations of 2500 economies of 100-quarter length. For the baseline (x symbols) we impose an uncertainty shock in the quarter labelled 1. For the uncertainty and TFP shock (► symbols), we also impose an aggregate productivity shock with average equal to -2%, allowing normal evolution of the economy afterwards. Clockwise from the top left, we plot the percent deviations of cross-economy average output, labor, consumption, and investment from their values in quarter 0.
Figure 9: The impact of an uncertainty shock is robust to a wide range of alternative calibrations

Notes: Based on independent simulations of 2500 economies of 100-quarter length. For all simulations we impose an uncertainty shock in the quarter labelled 1, allowing normal evolution of the economy afterwards. Baseline (x symbols) is the estimated baseline path. Other paths plot responses assuming a reduction in low-uncertainty micro volatility ($\sigma^2_{Z_L}$, o symbols), the high-uncertainty increase in micro volatility ($\sigma^2_{Z_H}/\sigma^2_{Z_L}$, diamonds), low-uncertainty macro volatility ($\sigma^a_{A_L}$, + symbols), the high-uncertainty increase in macro volatility ($\sigma^a_{A_H}/\sigma^a_{A_L}$, * symbols), the frequency of an uncertainty shock ($\pi_{L,H}$, stars), the persistence of an uncertainty shock ($\pi^0_{L,H}$, ▶ symbols), and all adjustment costs for labor and capital (▲ symbols). Clockwise from the top left, we plot the percent deviations of cross-economy average output, labor, consumption, and investment from their values in quarter 0.
Figure 10: The impact of an uncertainty shock combines Oi-Hartman-Abel, real options & consumption smoothing effects

Notes: Based on independent simulations of 2500 economies of 100-quarter length. For all simulations we impose an uncertainty shock in the quarter labelled 1, allowing normal evolution of the economy afterwards. GE, adjustment costs (x symbols) is the baseline, and the PE responses are partial equilibrium paths with adjustment costs (+ symbols) and without adjustment costs (o symbols). Clockwise from the top left, we plot the percent deviations of cross-economy average output, labor, consumption, and investment from their values in quarter 0. Note that PE economies have no consumption concept, with deviations therefore set to 0.
Figure 11: The impact of an uncertainty shock is reduced by lower rates of capital depreciation or labor attrition

Notes: Based on independent simulations of 2500 economies of 100-quarter length. For all simulations we impose an uncertainty shock in the quarter labelled 1, allowing normal evolution of the economy afterwards. Baseline (x symbols) is the estimated baseline path. The two other paths plot responses assuming a 25% reduction in the capital depreciation rate (o symbols) and labor depreciation rate (+ symbols). Clockwise from the top left, we plot the percent deviations of cross-economy average output, labor, consumption, and investment from their values in quarter 0.
**Figure 12: Policy is less effective in the aftermath of an uncertainty shock**

Output Impact of a 1% Wage Subsidy (in percent from value with no subsidy)

- **Subsidy during normal times**: Stimulative effect declines by over two thirds

**Notes**: Based on independent simulations of 2500 economies of 100-quarter length. For a wage subsidy in normal times (black bar, left), we provide an unanticipated 1% wage bill subsidy to all firms in quarter 1, allowing the economy to evolve normally thereafter. We also simulate an economy with no wage subsidy in quarter 1. The bar height is the percentage difference between the cross-economy average subsidy and no subsidy output paths in quarter 1. For the wage subsidy with an uncertainty shock (red bar, right), we repeat the experiment but simultaneously impose an uncertainty shock in quarter 1.
A Online Appendix: Census Uncertainty Data

We use data from the Census of Manufactures (CM) and the Annual Survey of Manufactures (ASM) from the U.S. Census Bureau to construct an establishment-level panel. Using the Compustat-SSEL bridge (CPST-SSEL) we merge the establishment-level data with Compustat and CRSP high frequency firm-level financial and sales data which allows us to correlate firm and industry-level cross-sectional dispersion from Census data with stock returns volatility measures. For industry-level deflators, and to calculate production function elasticities, we use industry-level data from the NBER-CES productivity database, the Federal Reserve Board (prices and depreciation), the BLS (multifactor productivity) and the BEA (fixed assets tables). In this appendix we describe each of our data sources, the way we construct our samples, and the way each variable is constructed. In constructing the TFP variables we closely follow Syverson (2004).

A.1 Data Sources

A.1.1 Establishment Level

The establishment-level analysis uses the CM and the ASM data. The CM is conducted every 5 years (for years ending 2 and 7) since 1967 (another CM was conducted at 1963). It covers all establishments with one or more paid employees in the manufacturing sector (SIC 20-39 or NAICS 31-33) which amounts to 300,000 to 400,000 establishments per survey. Since the CM is conducted at the establishment-level, a firm which operates more than one establishment files a separate report for each establishment. As a unique establishment-level ID we use the LBDNUM variable which allows us to match establishments over time within the CM and between the CM and the ASM. We use the FIRMID variable to match establishments to the Compustat-SSEL bridge which allows us to match to Compustat and CRSP firm’s data using the Compustat CUSIP identifier.

For annual frequency we add the ASM files to the CM files constructing a panel of establishments from 1972 to 2011 (using the LBDNUM identifier). Starting 1973, the ASM is conducted every year in which a CM is not conducted. The ASM covers all establishments which were recorded in the CM above a certain size and a sample of the smaller establishments. The ASM includes 50,000 to 75,000 observations per year. Both the CM and the ASM provide detailed data on sales, value added, labor inputs, labor cost, cost of materials, capital expenditures, inventories and more. We give more details on the variables we use in the variables construction subsection below.

A.1.2 Firm Level

We use Compustat and CRSP to calculate volatility of sales and returns at the firm level. The Compustat-SSEL bridge is used to match Census establishment data to Compustat and CRSP firm’s data using the Compustat CUSIP identifier. The bridge includes a mapping (m:m) between FIRMID (which can be found in the CM and ASM) and CUSIP8 (which can be found in Compustat and CRSP). The bridge covers the years 1976 to 2005. To extend the bridge to the entire sample of our analysis (1972-2010), we assigned each FIRMID after 2001 to the last observed CUSIP8 and before 1976 to the first observed CUSIP8.

From the CRSP data set we obtain daily and monthly returns at the firm level (RET). From Compustat we obtain firm-level quarterly sales (SALEQ) as well as data on equity (SEQQ) and debt (DLTTQ and DLCQ) which is used to construct the leverage ratio (in book values).

A.1.3 Industry Level

We use multiple sources of industry-level data for variables which do not exist at the establishment or firm level including price indices, cost shares, depreciation rates, market to book ratio of capital, price indices, cost shares, depreciation rates, market to book ratio of capital, price indices, cost shares, depreciation rates, market to book ratio of capital.

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8The 2010 and 2011 ASM data became available only at late stages of the project. To avoid repeating extensive census disclosure analysis, in Tables 2 and 3 we use data only up to 2009. The 2011 data became available only recently, therefore the SMM estimation uses data only up to 2010.

9The access to CRSP and Compustat data sets is through WRDS: https://wrds-web.wharton.upenn.edu/wrds/.

10We do this assignment for 2002-2005, since the bridge has many missing matches for these years.
import-export data and industrial production.

The NBER-CES Manufacturing Industry Database is the main source for industry-level price indices for total value of shipments (PISHIP), and capital expenditures (PIINV).\(^\text{11}\) It is also the main source for industry-level total cost of inputs for labor (PAY). The total cost variable is used in the construction of industry cost shares. We match the NBER data to the establishment data using 4-digit SIC87 codes for the years 1972-1996 and 6-digit NAICS codes starting 1997.\(^\text{12}\) We complete our set of price indices using FRB capital investment deflators, with separate deflators for equipment and structures, kindly provided to us by Randy Becker.

The BLS multifactor productivity database is used for constructing data on industry-level cost of capital and capital depreciation.\(^\text{13}\) In particular data from the tables “Capital Detail Data by Measure for Major Sectors” is used. From these tables we obtain data on depreciation rates (table 9a: EQDE, STDE), capital income (table 3a: EQKY STKY), productive capital (table 4a: EQPK, STPK), and an index of the ratio of capital input to productive stock (table 6b: EQKC, STKC). All measures are obtained separately for equipment and for structures (there are the EQ and ST prefix respectively). We use these measures to recover the cost of capital in production at the industry level. We match the BLS data to the establishment data using 2-digit SIC87 codes for the years 1972-1996 and 3 digit NAICS codes starting 1997.

We use the BEA fixed assets tables to transform establishment-level capital book value to market value. For historical cost we use tables 3.3E and 3.3S for equipment and for structures respectively.\(^\text{14}\) For current cost we use tables 3.1E and 3.1S.

The industrial production index constructed by the Board of Governors of the Federal Reserve System (FRB) is used to construct annual industry-level volatility measures.\(^\text{15}\) The data is at a monthly frequency and is provided at NAICS 3-digit to 6-digit level. We match the data to establishment-level data using the most detailed NAICS value available in the FRB data. Since ASM and CM records do not contain NAICS codes prior to 1997, we obtain for each establishment in our sample a modal NAICS code which will be non-missing in the case that the establishment appears for at least one year after 1996. For establishments who do not appear in our sample after 1996 we use an empirical SIC87-NAICS concordance. This concordance matches to each SIC87 code its modal NAICS code using establishments which appear in years prior to 1997 and after 1997.

A.2 Sample Selection

We have five main establishment samples which are used in our analysis of the manufacturing sector. The largest sample includes all establishments which appear in the CM or ASM for at least two consecutive years (implicitly implying that we must have at least one year from the ASM, therefore ASM sampling applies). In addition we exclude establishments which are not used in Census tabulation (TABBED=N), establishments with missing or nonpositive data on total value of shipments (TVS) and establishments with missing values for LBDNUM, value added (VA), labor inputs or investment. We also require each establishment to have at least one record of capital stock (at any year). This sample consists of 211,939 establishments and 1,365,759 establishment-year observations.

The second sample, which is our baseline sample, keeps establishments which appear for at least 25 years between 1972 and 2009. This sample consists of 15,673 establishments and 453,704 establishment-year observations.\(^\text{16}\)

The third sample we use is based on the baseline sample limited to establishments for which firms have CRSP and Compustat records, with nonmissing values for stock returns, sales, equity and debt. The sample includes 10,498 establishments with 172,074 establishment-year observations.

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\(^\text{11}\) See: http://www.nber.org/data/nbprod2005.html for the public version. We thank Wayne Gray for sharing his version of the dataset that is updated to 2009.

\(^\text{12}\) The NBER-CES data are available only through 2009. 2010 industry-level data are therefore projected using an AR(4) regression for all external datasets.

\(^\text{13}\) See http://www.bls.gov/mfp/mprdload.htm.

\(^\text{14}\) See http://www.bea.gov/national/FA2004/SelectTable.asp.


\(^\text{16}\) As the 2010 ASM data became available only very recently, whenever 2010 data is used we keep the sample of establishments unchanged. For example, we choose establishments that were active for 25 years between 1972 and 2009, but use data for these establishments also from 2010.
The fourth sample uses a balanced panel of establishments which were active for all years between 1972 and 2009. This sample consists of 3,449 establishments and 127,182 establishment-year observations.

Our last sample (used in Figures 1 and 2), is based on the first sample, but includes only establishments that were active in 2005, 2006, 2008 and 2009.

When calculating annual dispersion measures using CRSP and Compustat (see Table 1), we use the same sampling criteria as in the baseline ASM-CM sample, keeping only firms which appear for at least 25 years.

### A.3 Variable Construction

#### A.3.1 Value Added

We use the Census value added measure which is defined for establishment $j$ at year $t$ as

$$v_{j,t} = Q_{j,t} - M_{j,t} - E_{j,t},$$

where $Q_{j,t}$ is nominal output, $M_{j,t}$ is cost of materials and $E_{j,t}$ is cost of energy and fuels. Nominal output is calculated as the sum of total value of shipments and the change in inventory from previous year (both finished inventory and work in progress inventory).

In most of the analysis we use real value added. In this case, we deflate value added by the 4-digit industry price of shipment (PISHIP) given in the NBER-CES data set.

#### A.3.2 Labor Input

The CM and ASM report for each establishment the total employment (TE), the number of hours worked by production workers (PH), the total salaries for the establishment (SW) and the total salaries for production workers (WW). The surveys do not report the total hours for non-production workers. In addition, one might suspect that the effective unit of labor input is not the same for production and non-production workers. We calculate the following measure of labor inputs

$$n_{j,t} = \frac{SW_{j,t}}{WW_{j,t}} PH_{j,t}.$$

#### A.3.3 Capital Input

There are two issues to consider when constructing the capital measure. First, capital expenditures rather than capital stock are reported in most survey years, and when capital stock is reported it is sensitive to differences in accounting methods over the years. Second, the reported capital in the surveys is book value. We deal with the latter by first converting book to market value of capital stocks using BEA fixed asset tables which include both current and historical cost of equipment and structures stocks by industry-year. We address the first issue using the perpetual inventory method, calculating establishment-level series of capital stocks using the plant’s initial level of capital stock, the establishment-level investment data and industry-level depreciation rates. To apply the perpetual inventory method we first deflate the initial capital stock (in market value) as well as the investment series using FRB capital investment deflators. We than apply the formula $K_t = (1 - \delta_t) K_{t-1} + I_t$. This procedure is done separately for structures and for equipment. However, starting in 1997, the CM does not separately report capital stocks for equipment and structures. For plants which existed in 1992, we can use the investment data to back out capital stocks for equipment and structures separately after 1992. For plants born after 1992, we assign the share of capital stock to equipment and structures to match the share of investment in equipment and structures.

#### A.3.4 TFP and TFP Shocks

For establishment $j$ in industry $i$ at year $t$ we define value added based total factor productivity (TFP) $\bar{z}_{j,i,t}$ as

$$\bar{z}_{j,i,t} = \frac{Q_{j,t} - M_{j,t} - E_{j,t}}{E_{j,t}}.$$
\[
\log(\hat{z}_{j,i,t}) = \log(v_{j,i,t}) - \alpha_{i,t}^S \log(k_{j,i,t}^S) - \alpha_{i,t}^E \log(k_{j,i,t}^E) - \alpha_{i,t}^N \log(n_{j,i,t}),
\]

where \(v_{j,i,t}\) denotes value added (output less materials and energy inputs), \(k_{j,i,t}^S\) represents the capital stock of structures, \(k_{j,i,t}^E\) represents the capital stock of equipment and \(n_{j,i,t}\) the total hours worked as described above.

\(\alpha_{i,t}^S\), \(\alpha_{i,t}^E\), and \(\alpha_{i,t}^N\) are the cost shares for structures, equipment, and labor. These cost shares are recovered at 4-digit industry level by year, as is standard in the establishment productivity estimation literature (see, for example, the survey in Foster, Haltiwanger and Krizan, 2000). We generate the cost shares such that they sum to one. Define \(c_{i,t}^x\) as total cost of input \(x\) for industry \(i\) at year \(t\). Then for input \(x\)

\[
\alpha_{i,t}^x = \frac{c_{i,t}^x}{\sum_{x \in \{S, E, N\}} c_{i,t}^x}, \quad x = \{S, E, N\}.
\]

We use industry-level data to back out \(c_{i,t}^x\). The total cost of labor inputs \(c_{i,t}^N\) is taken from the NBER-CES Manufacturing Industry Database (PAY). The cost of capital (for equipment and structures) is set to be capital income at the industry level. The BLS productivity dataset includes data on capital income at the 2-digit industry level. To obtain capital income at 4-digit industry level we apply the ratio of capital income to capital input calculated using BLS data to the 4-digit NBER-CES capital data.

Given the cost shares, we can recover \(\log(\hat{z}_{j,i,t})\). We then define TFP shocks \((e_{j,i,t})\) as the residual from the following first-order autoregressive equation for establishment-level log TFP:

\[
\log(\hat{z}_{j,i,t}) = \rho \log(\hat{z}_{j,i,t-1}) + \mu_i + \lambda_t + e_{j,i,t},
\]

where \(\mu_i\) are plant fixed effects and \(\lambda_t\) are year dummies.

### A.3.5 Microeconomic Uncertainty Dispersion-Based Measures

Our main micro uncertainty measures are based on establishment-level TFP shocks \((e_{j,i,t})\) and on establishment-level growth in employment and in sales. For variable \(x\) we define establishment \(i\)’s growth rate for year \(t\) as \(\Delta x_{i,t} = (x_{i,t+1} - x_{i,t})/0.5 \times x_{i,t+1} + 0.5 \times x_{i,t}\).

Aggregate Level: In Table 1, to measure uncertainty at the aggregate level, we use the interquartile range (IQR) and the standard deviation of both TFP shocks and sales and employment growth by year. We consider additional measures for TFP shocks that allow for more flexibility in the AR regression (21) used to back out the shocks. In particular we report the dispersion of TFP shocks, which were calculated by running (21) at the 3-digit industry level (industry by industry), effectively allowing for \(\rho\) and for \(\lambda_t\) to vary by industry.

We use three additional aggregate uncertainty measures which are not based on Census data. We use CRSP to calculate the firms’ cross-sectional dispersion of monthly stock returns at a monthly frequency, and Compustat to calculate the cross-sectional dispersion of sales’ growth at a quarterly frequency, where sales growth is defined as \((x_{i,t+4} - x_{i,t})/(0.5 \times x_{i,t+4} + 0.5 \times x_{i,t})\). We use the industrial production index constructed by the FRB to calculate the cross-sectional dispersion of industry production growth \((x_{i,t+12} - x_{i,t})/(0.5 \times x_{i,t+12} + 0.5 \times x_{i,t})\) at the monthly level.

Firm Level: To measure uncertainty at the firm level, we use the weighted mean of the absolute value of TFP shocks and sales growth, where we use establishments’ total value of shipments as weights. As an example, the uncertainty measure for firm \(f\) at year \(t\) using TFP shocks is calculated as

\[
\frac{\sum_{j \in f} TVS_{j,t} |e_{j,t}|}{\sum_{j \in f} TVS_{j,t}},
\]

and it is calculated similarly for growth measures.

Industry Level: At the industry level we use both IQR (Table 2 and Table 3) and weighted mean of absolute values as uncertainty measures.
A.3.6 Micro Volatility Measures

Using CRSP, Compustat, and FRB data, we construct firm-level and industry-level annual volatility measures.

Firm Level: At the firm level we construct volatility measures using firms’ stock returns. We use standard deviation of daily and monthly returns over a year to generate the stock volatility of a firm. For the monthly returns we limit our samples to firms with data on at least 6 months of returns in a given calendar year. For monthly returns we winsorize records with daily returns which are higher or lower than 25%. As an alternative measure we follow Leahy and Whited (1996) and Bloom, Bond, and Van Reenen (2007) in implementing a leverage-adjusted volatility measure which eliminates the effect of gearing on the variability of stock returns. To generate this measure we multiply the standard deviation of returns for firm \( f \) at year \( t \) by the ratio of equity to (equity + debt), with equity measured using the book value of shares (SSEQ) and debt measured using the book value of debt (DLTTQ + DLCQ). To match the timing of the TFP shock in the regressions (calculated between \( t \) and \( t + 1 \), see (21)), we average over the standard deviation of returns at year \( t \) and the standard deviation at year \( t + 1 \).

For volatility of sales we use the standard deviation over a year of a firm’s annual growth calculated at a quarterly rate \( (x_{i,t+4} - x_{i,t})/(0.5 \times x_{i,t+4} + 0.5 \times x_{i,t}) \).

Industry Level: For industry level measures of volatility we use the standard deviation over a year of an industry’s annual growth calculated at a monthly rate \( (x_{i,t+12} - x_{i,t})/(0.5 \times x_{i,t+12} + 0.5 \times x_{i,t}) \) using the industrial production index constructed by the FRB.

A.3.7 Industry Characteristics

In Table 2 we use measures for industry business conditions and for industry characteristics. To proxy for industry business conditions we use either the mean or the median plant’s real sales growth rates within industry year. Industry characteristics are constant over time and are either level or dispersion measures. For levels we use medians, implying that a typical measure would look like

\[
\text{Median}_{j \in i} \left( \frac{1}{T} \sum_{t=1}^{T} x_{jt} \right),
\]

where \( x_{jt} \) is some characteristic of plant \( j \) at year \( t \) (e.g. plant total employment). The industry-level measure is calculated as the median over all plants in industry \( i \) of the within plant mean over time of \( x_{jt} \). The dispersion measures are similar but use the IQR instead of medians:

\[
\text{IQR}_{j \in i} \left( \frac{1}{T} \sum_{t=1}^{T} x_{jt} \right).
\]

One exception is the measure of industry geographic dispersion, which is calculated as the Ellison-Glaeser dispersion index at the county level.
In this computational appendix we first lay out the broad solution algorithm for the model, which follows the generalized Krusell and Smith (1998) approach as laid out by Khan and Thomas (2008). We also provide information on the practical numerical choices made in the implementation of that solution method. Then, we discuss the calculation of the approximate impulse response or effects of an uncertainty shock in this economy, together with the various other experiments analyzed in the main text. We conclude with a discussion of the accuracy of the forecast rule used to predict the evolution of the approximate aggregate state in the economy as well as other related solution techniques for this class of models.

As we discuss below (see the section titled "Alternative Forecasting or Market-Clearing Assumptions") we consider two alternatives to the basic forecasting rule discussed below. For each of these method we resolve the GE model until convergence of the forecasting rule is achieved. Importantly, as we discuss below the main results are robust across the different solution methods.

B.1 Solution Algorithm

The marginal-utility transformed Bellman equation describing the firm problem in the main text is reproduced below and results in a convenient problem with constant discounting at rate $\beta$ but a transformation of period dividends by a marginal utility price $p$:

$$
\hat{V}(k, n-1, z; A, \sigma^A, \sigma^Z, \mu) = \max_{\{i,n\}} \left\{ p(A, \sigma^A, \sigma^Z, \mu) \left( y - w(A, \sigma^A, \sigma^Z, \mu) n - i \right) + \beta \mathbb{E} \left[ \hat{V}(k', n', z'; A', \sigma^{A'}, \sigma^{Z'}, \mu') \right] \right\}.
$$

We can see that the aggregate state of the economy is $(A, \sigma^A, \sigma^Z, \mu)$ and as outlined in the main text household optimization implies the relationships

$$
p(A, \sigma^A, \sigma^Z, \mu) = C(A, \sigma^A, \sigma^Z, \mu)^{-\eta}
$$

$$
w(A, \sigma^A, \sigma^Z, \mu) = \theta N(A, \sigma^A, \sigma^Z, \mu)^{\chi-1} C(A, \sigma^A, \sigma^Z, \mu)^{\eta}.
$$

The calibration we choose, with log consumption utility $(\eta = 1)$ and an infinite Frisch elasticity of substitution $(\chi = 1)$, implies that the equilibrium relationships are given by

$$
p(A, \sigma^A, \sigma^Z, \mu) = C(A, \sigma^A, \sigma^Z, \mu)^{-1}
$$

$$
w(A, \sigma^A, \sigma^Z, \mu) = \theta C(A, \sigma^A, \sigma^Z, \mu) = \frac{\theta}{p(A, \sigma^A, \sigma^Z, \mu)}.
$$

With these choices wages $w$ are a function of the marginal utility price $p$, so the evolution of the aggregate equilibrium of the economy is fully characterized by the following two mappings:

$$
p = \Gamma_p(A, \sigma^A, \sigma^Z, \mu)
$$

$$
\mu' = \Gamma_\mu(A, \sigma^A, \sigma^Z, \mu).
$$

There are three related but distinct computational challenges involved in the characterization of the mappings $\Gamma_p$ and $\Gamma_\mu$. First, the cross-sectional distribution $\mu$ is generally intractable as a state variable. Second, the number of aggregate state variables (excluding the cross-sectional distribution $\mu$) is large, since not only aggregate productivity $A$ but also the levels of macro $\sigma^A$ and micro $\sigma^Z$ uncertainty are included. Third, the equilibrium price mapping $\Gamma_p$ must be computed or approximated in some fashion.

We address each of these challenges in the following fashion. As noted in the main text, we assume that a single two-state Markov process $S \in \{0, 1\}$ for uncertainty governs the
evolution of micro and macro volatility across two discrete levels each, so that
\[ S = 0 \rightarrow \sigma^A = \sigma^A_L, \sigma^Z = \sigma^Z_L \]
\[ S = 1 \rightarrow \sigma^A = \sigma^A_H, \sigma^Z = \sigma^Z_H, \]
where the transition matrix for \( S \) is governed by
\[ \Pi^S = \begin{bmatrix} 1 - \pi_{L,H} & \pi_{L,H} \\ 1 - \pi_{H,H} & \pi_{H,H} \end{bmatrix}. \]
We then also approximate the intractable cross-sectional distribution \( \mu \) in the aggregate state space with the current aggregate capital level \( K = \int k(z, k, n-1) d\mu \) as well as the lagged uncertainty state \( S_{-1} \). The approximate aggregate state vector is now given by \((A, S, S_{-1}, K)\), which addresses the first and second computational challenges outlined above.

We are now in a position to define tractable approximations to the equilibrium mappings \( \Gamma_p \) and \( \Gamma_\mu \) using the following log-linear rules \( \hat{\Gamma}_p \) and \( \hat{\Gamma}_K \):
\[ \hat{\Gamma}_p : \log(\hat{p}) = \alpha_p (A, S, S_{-1}) + \beta_p (A, S, S_{-1}) \log(K) \]
\[ \hat{\Gamma}_K : \log(\hat{K}') = \alpha_K (A, S, S_{-1}) + \beta_K (A, S, S_{-1}) \log(K). \]

The approximations of the aggregate state space and the explicit forms chosen for the equilibrium mappings laid out above are assumptions which can and must be tested for internal accuracy. Below, we will calculate and discuss a range of accuracy statistics commonly used in the literature on heterogeneous agents models with aggregate uncertainty. Although this specification with lagged uncertainty \( S_{-1} \) governing coefficients in the forecast rule serves as our baseline, we also consider below two extensions to even larger forecast rule systems below, with additional endogenous aggregate state variables or a longer series of lagged uncertainty realizations included within the forecast rule. None of these extensions changes the impact of an uncertainty shock on economic output much at all relative to our baseline, uniformly causing a recession of around 2.5%.

However, we can now lay out the approximated firm Bellman equation as \( \hat{V} \), where
\[ \hat{V}(k, n-1, z; A, S, S_{-1}, K) = \]
\[ \max_{\{i,n\}} \left\{ \hat{p}(A, S, S_{-1}, K) \left( y - \frac{\theta}{\hat{p}(A, S, S_{-1}, K)} n - i - ACk - ACn \right) + \beta \mathbb{E} \left[ \hat{V}(k', n', z'; A', S', S', K') \right] \right\}. \]

Given this approximated equilibrium concept, we can now lay out the solution algorithm. First, initialize the equilibrium mappings or forecast rules \( \hat{\Gamma}_p^{(1)} \) and \( \hat{\Gamma}_K^{(1)} \) by guessing initial coefficients \( \left( \alpha_p^{(1)} (A, S, S_{-1}), \beta_p^{(1)} (A, S, S_{-1}) \right) \) and \( \left( \alpha_K^{(1)} (A, S, S_{-1}), \beta_K^{(1)} (A, S, S_{-1}) \right) \). Then, perform the following steps in iteration \( q = 1, 2, \ldots \) of the algorithm:

**Step 1 - Firm Problem Solution**

Solve the idiosyncratic firm problem laid out in the Bellman equation for \( \hat{V} \) above, conditional upon \( \hat{\Gamma}_p^{(q)} \) and \( \hat{\Gamma}_K^{(q)} \), resulting in an approximated firm value function \( \hat{V}^{(q)} \).

**Step 2 - Unconditional Simulation of the Model**

Based upon the approximated firm value \( \hat{V}^{(q)} \), simulate the economy unconditionally for \( T \) periods, without imposing adherence to the assumed equilibrium price mapping \( \hat{\Gamma}_p^{(q)} \).

**Step 3 - Equilibrium Mapping Update**

Based upon the simulated data from the model computed in Step 2, update the forecast mappings to obtain \( \hat{\Gamma}_p^{(q+1)} \) and \( \hat{\Gamma}_K^{(q+1)} \).

**Step 4 - Test for Convergence**

If the approximate mappings have converged, i.e. if the difference between \( \hat{\Gamma}_p^{(q+1)} \) and \( \hat{\Gamma}_K^{(q+1)} \) is smaller than some tolerance \( \varepsilon_\Gamma \) according to some predetermined criterion, exit the algo-
If the mappings have not converged, return to Step 1 for the $q+1$-th iteration.

The practical numerical implementation of each of the Steps 1-4 laid out above requires some explanation in more detail. We discuss each step in turn now, noting that a pack containing all of the code for this paper can be found on Nicholas Bloom’s website.

**Firm Problem Solution**

We discretize the state space of the idiosyncratic firm problem. For the endogenous idiosyncratic variables $k$ and $n$, we choose log-linear grids of size $n_k = 91$ and $n_n = 37$ closed with respect to depreciation. For the exogenous productivity processes $z$ and $A$, we discretize each process following a straightforward generalization of Tauchen (1986) to the case of time-varying volatility, resulting in processes with $n_z = n_A = 5$ discrete productivity points. Given the discretization of the firm problem, we compute $\hat{V}(q)$ using Howard policy acceleration with 200 value function update steps within each policy loop until the policy functions converge to within some prescribed tolerance. This routine is described in, for example, Judd (1998). Here, the discrete nature of the firm problem allows for exact convergence of firm policies. Throughout the problem, continuation values are computed using linear interpolation in the forecast value of aggregate capital $\hat{K}$ implied by the mapping $\hat{F}_K^{(q)}$.

**Unconditional Simulation of the Model & Convexified Market Clearing**

We simulate the model using a fixed set of $T = 5000$ periods of exogenous aggregate productivity and uncertainty realizations $(A_t, S_t)$, $t = 1,...,T$ which follow the discrete Markov chain structures discussed above and are drawn once and for all outside the solution algorithm’s outer loop (Steps 1-4).

Within the solution loop, we follow the nonstochastic or histogram-based approach to tracking the cross-sectional distribution proposed by Young (2010). This simulation approach avoids the Monte Carlo sampling error associated with the simulation of individual firms and is faster in practice. In particular, we track a histogram $\hat{\mu}_t$ of weights on individual points $(k, n_{-1}, z)$ in the firm-level discretized state space from period to period. Let the policy functions in period $t$ be given by $n_t(k, n_{-1}, z)$ and $k_t'(k, n_{-1}, z)$ and the transition matrix over idiosyncratic productivity in period $t$ be given by $\Pi^Z(z, z'; S_t)$. If we consider discretized triplets $(k, n_{-1}, z)_i$, $i = 1,...,n_kn_{-1}n_z$ then we have that $\mu_{t+1}$ is determined by

$$\mu_{t+1}((k', n, z')_j) = \sum_{(k, n, z)_i} \mu_t((k, n, z)_i) \Pi^Z(z_i, z'_j; S_t) \mathbb{I}(k_j = k'_t((k, n, z)_i), n_j = n_t((k, n, z)_i)).$$

The calculation of the individual firm policy functions $k_t'$ and $n_t$ in period $t$ must be consistent with market-clearing as well as individual firm optimization, however, in the sense that the simulated consumption level $C_t$ and hence market-clearing marginal utility price $p_t = \frac{1}{C_t}$ must be generated by the approximate cross-sectional distribution $\mu_t$ as well as the firm policy rules. To guarantee that this occurs, in each period we must iterate over a market clearing price $p$, using the continuation value $\hat{V}(q)$ but discarding the forecast price level $\tilde{p}_t^{(q)}$. For any guess $\tilde{p}$, we re-optimize the right hand side of the firm Bellman equation given $\tilde{p}$, i.e. we compute firm policy functions $k_t'$ and $n_t$ to solve

$$\max_{k',n} \left\{ \tilde{p} \left( y - \frac{\theta}{p'} n - i - AC^k - AC^n \right) + \beta \mathbb{E} \left[ \hat{V}(q)(k', n, z', A', S', S, \hat{K}(q)) \right] \right\}.$$

Market clearing occurs when the consumption level implied by the price, $C(\hat{p})$ is equal to the reciprocal of that price, $\frac{1}{\hat{p}}$. However, as mentioned above we employ a discretization method to the solution of individual firm problems. Although the discretization method is both fast and robust, the resulting excess demand function $\epsilon(\hat{p})$ may contain discontinuities associated with some positive mass of firms discretely shifting capital or investment policies in response to a small price shift. Any such discontinuities in the resulting excess demand function would remove our ability to guarantee arbitrarily precise market clearing. We directly address this issue in a manner which implies internally consistent aggregate dynamics, through “convexification” of firm policies and hence the underlying excess demand function. For
each period in our simulation of the model, we employ the following process to guarantee market clearing.

**Step 1 - Precompute certain values over a price grid**

Utilize a pre-determined grid of marginal utility price candidates of size $N_p, \{\tilde{p}_i\}_{i=1}^{N_p}$. For each individual candidate price level, recompute firm policy functions as discussed above, to obtain values $C(\tilde{p}_i)$ and $K'(\tilde{p}_i)$ defined as

$$C(\tilde{p}_i) = \sum_{(k,n-1,z)_i} \mu_i ((k,n-1,z)_i) (y - k' + (1- \delta_k)k - AC^k - AC^n)$$

$$K'(\tilde{p}_i) = \sum_{(k,n-1,z)_i} \mu_i ((k,n-1,z)_i) h'$$

We also label the value of the cross-sectional distribution over values of firm capital, labor, and idiosyncratic productivity in the next period that would prevail given the firm policies implied by the candidate value $\tilde{p}_i$ today as $\mu'(\tilde{p}_i)$.

**Step 2 - Construct the implied convexified excess demand function**

Given the set of candidate price and implied consumption values $\{\tilde{p}_i, C(\tilde{p}_i)\}_{i=1}^{N_p}$, linearly interpolate the consumption function to obtain the continuous piecewise linear approximating consumption function $C(\tilde{p})$ for aggregate consumption, which is defined for any candidate input price $\tilde{p}$ including those outside the initial grid. Then, define the convexified excess demand function

$$e(\tilde{p}) = \frac{1}{\tilde{p}} - \hat{C}(\tilde{p}),$$

which is both defined for arbitrary positive candidate input prices $\tilde{p}$. This convexified excess demand function is also continuous over its entire domain because both of the functions $\frac{1}{\tilde{p}}$ and $\hat{C}(\tilde{p})$ are continuous. In practice, the excess demand function computed in this manner is also strictly decreasing. Intuitively, the process of linear interpolation of implied consumption values over a pre-determined grid to compute excess demand “convexifies” the market clearing process.

**Step 3 - Clear markets with arbitrary accuracy**

Given a continuous, strictly decreasing excess demand function, we use a robust hybrid bisection/inverse quadratic interpolation algorithm in $\tilde{p}$ in each period to solve the clearing equation

$$\tilde{e}(\tilde{p}) = 0$$

to any desired arbitrary accuracy $\varepsilon_p$.

**Step 4 - Update the firm distribution and aggregate transitions**

With a market clearing price $p^*$ in hand for the current period $t$, we have now cleared an approximation to the excess demand function. However, we must then update the underlying firm distribution and aggregate transitions in a manner which is consistent with the construction of the approximated excess demand function. In particular, we do so by first computing the underlying linear-interpolation weights in the approximation $\hat{C}(\tilde{p})$ at the point $p^*$. In other words, we compute the value

$$\omega(p^*) = \frac{p^* - \tilde{p}_{i^*}}{\tilde{p}_{i^*} - \tilde{p}_{i^* - 1}},$$

where $[\tilde{p}_{i^* - 1}, \tilde{p}_{i^*}]$ is the nearest bracketing interval for $p^*$ on the grid $\{\tilde{p}_i\}_{i=1}^{N_p}$. Note that $\omega(p^*)$ defined in this manner lies in the interval $[0,1]$, and also note by the definition of linear interpolation that

$$\hat{C}(p^*) = (1 - \omega(p^*))C(\tilde{p}_{i^* - 1}) + \omega(p^*)C(\tilde{p}_{i^*}).$$

For each endpoint of the interval $\tilde{p}_{i^* - 1}$ and $\tilde{p}_{i^*}$, we already have in hand values of capital next period $K'(\tilde{p}_{i^* - 1})$ and $K'(\tilde{p}_{i^*})$ as well as distributions next period $\mu'(\tilde{p}_{i^* - 1})$ and $\mu'(\tilde{p}_{i^*})$ which would prevail at the candidate prices. We simply update the cross-sectional
distribution $\mu_{t+1}$ and aggregate capital $K_{t+1}$ for the next period as

\[
K_{t+1} = (1 - \omega(p^*) )K'_{t}(\tilde{p}_{t-1}) + \omega(p^*)K'(\tilde{p}_{t})
\]

\[
\mu_{t+1} = (1 - \omega(p^*) )\mu'(\tilde{p}_{t-1}) + \omega(p^*)\mu'(\tilde{p}_{t}).
\]

In practice, when applying this market clearing algorithm, we use a clearing error tolerance of $\varepsilon_p = 0.0001$, therefore requiring that the maximum percentage clearing error cannot rise above 0.01% in any period. We also utilize a grid of size $N_p = 25$, after ensuring that a grid of this size delivers results which do not change meaningfully at higher grid densities. The resulting path of aggregate consumption and implied clearing errors - all of course within the required percentage band of less than 0.01% - are plotted over a representative portion of the unconditional simulation of our model in Figure B1.

One final computational choice deserves mention here. The simulation step, and in particular the determination of the market-clearing price $p_t$ in each period, is the most time-consuming portion of the solution algorithm, especially since firm policies and aggregate variables must be pre-computed along a grid of $N_p$ candidate price values. We have found that substantial speed gains can be obtained, with little change in the resulting simulated aggregate series, if the re-optimization of firm policies conditional upon a price for $\bar{p}$ in period $t$ is only computed for states above a certain weight $\varepsilon_{dist} = 0.0001$ in the histogram $\mu_t$. Thereafter, only those firm-level states $(k, n_{-1}, z)_i$ with weight above $\varepsilon_{dist}$ are used in the calculation of the market clearing price $p_t$ and the evolution of aggregates in the economy from $t$ to $t + 1$.

**Equilibrium Mapping Update**

At this point in the solution algorithm in iteration $q$ we have obtained a series of prices and capital stocks $(p_t, K_t)$, $t = 1, ..., T$ together with exogenous aggregate series $(A_t, S_t, S_{t-1})$. Recall that we set $T = 5000$. These simulated series are conditioned upon the equilibrium mappings $\hat{\Gamma}_p^{(q)}$ and $\hat{\Gamma}_k^{(q)}$. To update the equilibrium mappings, which are simply lists of coefficient pairs $\left( \alpha_p^{(q)} \left( (A, S, S_{-1})_i \right), \beta_p^{(q)} \left( (A, S, S_{-1})_i \right) \right)$ for each discrete triplet $(A, S, S_{-1})_i$, we first discard the $T_{erg} = 500$ initial periods in the simulation to remove the influence of initial conditions. For each set of values $(A, S, S_{-1})_i$, we collect the subset of periods $t \in \{T_{erg} + 1, ..., T\}$ with those particular exogenous aggregate states. We then update the mapping coefficients via the following OLS regressions on that subset of simulated data:

\[
\log(p_t) = \alpha_p((A, S, S_{-1})_i) + \beta_p((A, S, S_{-1})_i) \log(K_t)
\]

\[
\log(K_{t+1}) = \alpha_K((A, S, S_{-1})_i) + \beta_K((A, S, S_{-1})_i) \log(K_t).
\]

After collecting the estimated coefficients we obtain updated mappings $\hat{\Gamma}_p^{(q+1)}$ and $\hat{\Gamma}_K^{(q+1)}$.

**Test for Convergence**

At this point within the solution algorithm, there are several potential criteria which could in principle be used to determine final convergence of the equilibrium mappings and therefore the model solution. One option is to declare convergence if the maximum absolute difference between any two corresponding coefficients is less than some tolerance $\varepsilon_{mapping}$.

However, we have found that there are substantial speed gains, with little difference in the resulting simulated series, if instead convergence is defined based upon the accuracy of the forecast system itself. In particular, a commonly accepted practice in the literature is to define the internal accuracy of a forecast mapping based upon the maximum Den Haan (2010) statistics. These statistics for both capital $K$ and price $p$, which we label $DH_{K}^{max}$ and $DH_{p}^{max}$, respectively, are the maximum absolute log difference across the full model simulation of the simulated series $(p_t, K_t)$ and their dynamically forecasted counterparts $(\hat{p}_t^{DH}, K_t^{DH})$. “Dynamic forecasts” are simply the result of repeated application of the equilibrium mappings $\hat{\Gamma}_p$ and $\hat{\Gamma}_K$ using previously predicted values as explanatory or right-hand-side variables in the forward substitution. Such an approach allows for the accumulation of prediction error within the system to a more stringent degree than would result from a one-period or “static” evaluation of prediction errors. We conclude that the forecast mapping has converged, and therefore that the model has been solved, when the change
in the model’s accuracy statistics is less than a prescribed tolerance $\varepsilon_{\text{mapping}} = 0.01\%$, i.e. when
\[
\max\{|DH^{\text{max},(q+1)}_p - DH^{\text{max},(q)}_p|, |DH^{\text{max},(q+1)}_K - DH^{\text{max},(q)}_K|\} < \varepsilon_{\text{mapping}}.
\]

B.2 Conditional Response Calculations

In this subsection we describe the calculation of the response of the economy to an uncertainty shock, an uncertainty shock coincident with an aggregate productivity shock, and a policy or wage subsidy experiment.

**The Effects of an Uncertainty Shock**

Armed with the solution to the model following the algorithm outlined above, we compute the conditional response of the economy to an uncertainty shock by simulating $N = 2500$ independent economies of length $T_{IRF} = 100$. We designate $T_{\text{shock}} = 50$ as the shock period. In each economy $i$, we simulate the model as normal for periods $t = 1, \ldots, T_{\text{shock}} - 1$ starting from some initial conditions and following the procedure for within-period market clearing outlined above. In period $T_{\text{shock}}$, we impose high uncertainty for economy $i$, i.e. $S^i_{T_{\text{shock}}} = 1$. Thereafter, each economy $i$ evolves normally for periods $t = T_{\text{shock}} + 1, \ldots, T_{IRF}$.

Given any aggregate series of interest $X$, with simulated value $X^i_t$ in economy $i$, period $t$, we define the period $t$ response of the economy to an uncertainty shock in period $T_{\text{shock}}$ as
\[
\hat{X}_t = 100 \log \left( \frac{\bar{X}_t}{X^i_{T_{\text{shock}} - 1}} \right),
\]
where $\bar{X}_t$ is the cross-economy average level in period $t$: $\frac{1}{N} \sum_i X^i_t$. The notation $\hat{X}_t$ for the percentage deviations of a series from its pre-shock level will be used throughout this subsection in the context of various experiments and shocks. Also, note that for the purposes of labelling the figures in the main text, we normalize $T_{\text{shock}} = 1$. The initial conditions we use to start each simulation include a low uncertainty state, the median aggregate productivity state, and the cross-sectional distribution from a representative period in the unconditional simulation of the model.

**The Effects of an Uncertainty Shock and First-Moment Shock**

To simulate the effect of an uncertainty shock coincident with a first-moment shock to exogenous aggregate productivity $A$, we follow the same basic procedure as above, simulating independent economies $i = 1, \ldots, N$ for quarters $t = 1, \ldots, T_{IRF}$, where $N = 2500$ and $T_{IRF} = 100$. In the shock period, we impose a high uncertainty state for all economies, i.e. we set $S^i_{T_{\text{shock}}} = 1$ for all $i$. However, we also wish to impose a negative aggregate productivity shock in period $T_{\text{shock}}$ equal to -2% on average. To operationalize this, we choose a threshold probability $\xi$ and then draw independent uniform random variables $\xi^i \sim U(0, 1)$ for each economy $i$. With probability $\xi^i$, i.e. if $\xi^i \leq \tilde{\xi}$, we set the discretized aggregate productivity state $A^i_{T_{\text{shock}}}$ equal to the lowest grid point value. With probability $1 - \tilde{\xi}$, i.e. if $\xi^i > \tilde{\xi}$, we allow the aggregate productivity process to evolve normally for economy $i$ in period $T_{\text{shock}}$. For all economies, post-shock periods $t = T_{\text{shock}} + 1, \ldots, T_{IRF}$ evolve normally. By iterating on the value of $\tilde{\xi}$, we choose a probability of a first-moment shock which guarantees that, on average, a -2% shock obtains:
\[
\tilde{A}_{T_{\text{shock}}} = 100 \log \left( \frac{\tilde{A}^i_{T_{\text{shock}}}}{\tilde{A}^i_{T_{\text{shock}} - 1}} \right) = -2.
\]
Note that the plotted series of shock responses $\hat{X}_t$ for individual aggregate series of interest $X$ are defined exactly as in the baseline uncertainty shock case above, again normalizing the shock period to $T_{\text{shock}} = 1$ for plotting purposes.

**The Effects of a Wage Subsidy Policy Experiment**

The effect of a wage subsidy policy experiment require the simulation of three additional experiments, each with the same structure of $i = 1, \ldots, N$ independent economies with
quarters \( t = 1, \ldots, T_{IRF} \) where \( N = 2500 \) and \( T_{IRF} = 100 \). All shocks and policies are imposed in period \( T_{\text{shock}} = 50 \), with normal evolution until that period.

The first additional experiment we run is a wage subsidy with no uncertainty shock. In this case in period \( T_{\text{shock}} \) we impose an unanticipated wage bill subsidy so that the wage rate faced by firms is equal to \((100-1)\%\) of the equilibrium wage rate \( w \). This subsidy is financed with lump-sum taxes on the representative household. Afterwards each economy evolves normally, and we denote the resulting percentage deviations from the pre-shock period of this post-subsidy path by \( \tilde{X}_{t,\text{subsidy,normal}} \).

The second additional experiment we run is a wage subsidy concurrent with an uncertainty shock. In this case, in period \( T_{\text{shock}} \) we impose the same unanticipated wage bill subsidy but also a high uncertainty state \( S_{T_{\text{shock}}} = 1 \). Afterwards each economy evolves normally, and we denote the resulting post-subsidy path of this simulation by \( \tilde{X}_{t,\text{subsidy,unc}} \).

The third additional experiment we run is a simulation of “normal times,” where no wage subsidy and no uncertainty shock is imposed in period \( T_{\text{shock}} \). We denote the resulting path of this economy as \( X_{t,\text{normal}} \).

If we denote the baseline response of the economy to an uncertainty shock as \( \tilde{X}_{\text{unc}} \), then we can now define the two objects reported in the main text. The effective of a wage subsidy in normal times is simply
\[
\tilde{X}_{t,\text{subsidy,normal}} - \tilde{X}_{t,\text{normal}},
\]
while the effect of a wage subsidy with an uncertainty shock is
\[
\tilde{X}_{t,\text{subsidy,unc}} - \tilde{X}_{t,\text{unc}}.
\]

**An Alternative Approach to Computing the Effect of Uncertainty**

In nonlinear models there is not a universally defined notion of a conditional response or impulse response. As an alternative to our baseline method of computing the impact of uncertainty on the economy described above, we also have checked and report in Figure B4 the results from an alternative procedure constructed for nonlinear models based on Koop, et al. (1996). That procedure works as follows. Draw \( i = 1, \ldots, N \) sets of exogenous random draws for the uncertainty and aggregate productivity processes, each of which include \( t = 1, \ldots, T_{IRF} \) periods. Then, for each set of draws or economies \( i \), simulate two versions, a shock and no shock economy. In the shock economy, simulate all macro aggregates \( X_{it,\text{shock}} \) for each period \( t = 1, \ldots, T_{\text{shock}} - 1 \) as normal. Then, in period \( T_{\text{shock}} \) impose high uncertainty. For all periods \( t = T_{\text{shock}} + 1, \ldots, T_{IRF} \), simulate the economy as normal. Then, for version no shock, simulate all macro aggregates \( X_{it,\text{noshock}} \) unconditionally without any restrictions. The only difference in the exogenous shocks in the shock and noshock economies is the imposition of the single high uncertainty state in period \( T_{\text{shock}} \). The resulting effect of an uncertainty shock on the aggregate \( X \) is given by the cross-economy average percentage difference between the shock and no shock versions:
\[
\tilde{X}_{t,Koop} = \frac{1}{N} \sum_{i=1}^{N} \log \left( \frac{X_{it,\text{shock}}}{X_{it,\text{noshock}}} \right).
\]
By construction, this response is equal to zero before \( T_{\text{shock}} \). The figures plotted in the text normalize \( T_{\text{shock}} \) to 1 for labelling and report 100\( \tilde{X}_{t,Koop} \). Note that \( N, T_{IRF} \), and \( T_{\text{shock}} \) are identical to our baseline version. Figure B4 reports the response of output, labor, investment, and consumption to an uncertainty shock, computed in both the baseline and the alternative fashion described here, labeled “simulation differencing” in the figure. The response of the economy to an uncertainty shock is distinct but does not vary qualitatively across the two alternative notions of impulse responses.

**A Denser Grid to Solve the Model**

In our baseline solution to the model we, use \( n_z = n_A = 5 \) discrete productivity points for both micro productivity \( z \) and macro productivity \( A \). To explore the impact of the grid density on our conclusions about the impact of uncertainty on the macroeconomy, we also solved the model with a denser grid of \( n_z = n_A = 9 \) grids points for each productivity process. The move expands the size the numerical grid by a factor of \( 81/25 = 3.24 \). This
of course entails additional costs in the solution of the firm problem, but the unconditional simulation of the model had to be extended from \( T = 5,000 \) quarters to \( T = 15,000 \) quarters in order to ensure that the forecast rule for each configuration of aggregate productivity and uncertainty states had enough density to be accurately estimated. Figure B5 reports the resulting response of the economy to an uncertainty shock, computed using our baseline impulse response simulation method. The response of the economy to an uncertainty shock is distinct but does not vary qualitatively with the larger grid. Ideally the response could be computed with an even denser grid, although current computational limits render such an approach infeasible.

### B.3 Internal Accuracy of the Approximate Equilibrium Mappings

In this subsection we first report basic accuracy statistics emphasized by the literature on heterogeneous agents business cycle models for evaluation of the approximate equilibrium mappings or forecast rules \( \hat{\Gamma}_K \) and \( \hat{\Gamma}_p \). We also introduce and examine the results implied by alternative forecasting systems with more complexity than our baseline approach. Finally, we conclude with a brief discussion of alternative solution algorithms to Krusell Smith which have been proposed in the literature on heterogeneous agents business cycle models.

#### Static Accuracy Statistics

Recall that the prediction rules for current-period market-clearing prices \( \hat{\Gamma}_p \) and next period’s aggregate capital state take the following form:

\[
\log(\hat{p}_t) = \alpha_p(A_t, S_t, S_{t-1}) + \beta_p(A_t, S_t, S_{t-1}) \log(K_t)
\]

\[
\log(\hat{K}_{t+1}) = \alpha_K(A_t, S_t, S_{t-1}) + \beta_K(A_t, S_t, S_{t-1}) \log(K_t).
\]

Using the standard terminology of time series forecasting, we refer to the predicted price and capital levels \( \hat{p}_t \) and \( \hat{K}_{t+1} \) as “static” or one-period ahead forecasts, since their calculation exploits information from the current period \( t \), namely \( K_t \) and the state \( (A_t, S_t, S_{t-1}) \). For each discretized triplet \( (A, S, S_{-1}) \) of exogenous aggregate states, Table B1 reports the accuracy of these static forecasts for our baseline model based on two common metrics: the \( R^2 \) of each prediction regression and the percentage root mean squared error (RMSE) of the predictions. As the table shows, by these metrics the prediction rules are generally quite accurate with \( R^2 \) near 1 and RMSEs never rising above approximately 0.5%.

#### Alternative Forecasting or Market-Clearing Assumptions

The dynamics of consumption \( C \), and hence the dynamics of the market-clearing marginal utility price \( p = \frac{1}{C}, \) crucially influence the behavior of investment in our general equilibrium economy. A high price today relative to the future signals that investment is costly, and vice-versa, resulting in smoother investment paths relative to a partial equilibrium structure. Given the importance of the price \( p \) for determining investment dynamics, we now compare the behavior of output, together with realized and forecast prices, for four different economies that encompass different forecast rules or market-clearing assumptions for the aggregate price level \( p \). Dynamic forecast accuracy statistics for each approach are reported in Table B2, and Figure B2 plots the response of output, consumption, and the implied forecast errors for each strategy after an uncertainty shock.

1) **Baseline Economy**

This economy serves as the baseline for calculations of the uncertainty impulse responses presented in the main text. The forecast rule with lagged uncertainty included and the market clearing algorithm based on convexified excess demand are as described above.

2) **Extra Uncertainty Lags Economy**

This economy uses an identical market clearing algorithm as the baseline. However, the forecast levels of price and future capital are based on the generalized log-linear forecast rules

\[
\log(\hat{p}_t) = \alpha_p(A_t, S_t, S_{t-1}, ..., S_{t-k}) + \beta_p(A_t, S_t, S_{t-1}, ..., S_{t-k}) \log(K_t)
\]

\[
\log(\hat{K}_{t+1}) = \alpha_K(A_t, S_t, S_{t-1}, ..., S_{t-k}) + \beta_K(A_t, S_t, S_{t-1}, ..., S_{t-k}) \log(K_t).
\]
In other words, the forecast rule coefficients are allowed to depend upon a larger set of conditional variables with extra lags of uncertainty beyond $S_{t-1}$. In the results reported in Table B2 and Figure B2, the extra uncertainty lags results are computed with $k = 3$, i.e. with a total of 4 values or one year of uncertainty realizations $S_t, ..., S_{t-3}$ conditioned into the forecast rule. Because this procedure results in a smaller subsample for the regression update of each forecast rule, we also expand the size of the unconditional simulation of the model to 20,000 quarters rather than the 5,000 quarters used for the baseline model, continuing to discard the first 500 quarters to cleanse the simulation of the impact of initial conditions.

3) Extra Forecast Moment Economy

This economy adds an additional endogenous moment, $M_t$ to the forecast rules for marginal utility. The addition requires expansion of the baseline forecast rule system to include an additional explanatory variable on the right hand side as well as another equation, taking the resulting form

\[\log(\hat{p}_t) = \alpha_p (A_t, S_t, S_{t-1}) + \beta_p (A_t, S_t, S_{t-1}) \log(K_t) + \gamma_p (A_t, S_t, S_{t-1}) M_t\]

\[\log(\hat{K}_{t+1}) = \alpha_K (A_t, S_t, S_{t-1}) + \beta_K (A_t, S_t, S_{t-1}) \log(K_t) + \gamma_K (A_t, S_t, S_{t-1}) M_t\]

\[\hat{M}_{t+1} = \alpha_M (A_t, S_t, S_{t-1}) + \beta_M (A_t, S_t, S_{t-1}) \log(K_t) + \gamma_M (A_t, S_t, S_{t-1}) M_t\]

After the forecast system itself is expanded in the manner above, all value functions in the solution of the model must take another aggregate input $M_t$, but the computational strategy and market clearing approach based on convexified aggregate demand remain otherwise identical. We considered a number of candidate values for inclusion as the extra additional endogenous moment, including the second moments of idiosyncratic capital and labor inputs across firms. Our chosen moment, the predetermined beginning of period $t$ end of period $t-1$ cross-sectional covariance $M_t = Cov(\log k_{t-1}, \log z_{t-1})$, performs well among our candidates because it proxies for allocative efficiency in the economy as a measure of the alignment of idiosyncratic profitability and capital inputs across firms. After an uncertainty shock $M_t$ declines steadily, rising after uncertainty subsides. Within our forecast structure, the extra covariance moment $\hat{M}_t$ thus serves as a continuous proxy for “time since an uncertainty shock,” a useful piece of information given the rich dynamics of our economy. Because of the size of the expanded forecast rule in this case, we use an expanded unconditional simulation of 20,000 quarter length rather than the 5,000 quarters used to update the more parsimonious baseline forecast system.

4) Nonconvexified Clearing

As a comparison for economies 1)-3), this fourth alternative is based on approximate market clearing choosing prices in each period to come as close as possible to clearing the nonconvexified or raw excess demand function. As noted above, potential discontinuities due to discrete firm choices embedded in this excess demand function imply that markets may not be cleared to arbitrary accuracy, although the computational burden is substantially lower. Although the clearing algorithm is distinct, this economy uses a forecast system identical to the baseline economy.

How do each of these alternative computational strategies compare in practice? First, we evaluate their conditional patterns and accuracy after an uncertainty shock. The left panel of Figure B2 plots the response of output in each economy to an uncertainty shock, the middle panel plots the response of consumption, and the right panel plots the response of forecast errors, (the difference between the paths for realized prices $p_t$ and forecast prices $\hat{p}_t$). Crucially, each economy considered here delivers an immediate drop in output of around 2.5% in the wake of an uncertainty shock, followed by a quick recovery. Each of the economies relying upon arbitrarily accurate market clearing, i.e. the baseline economy (x symbols) the extra uncertainty lags economy (* symbols), and the economy with an extra forecast rule moment (diamonds), recovers more strongly than the economy relying on nonconvexified market clearing (o symbols). In every economy, the combination of rising consumption levels and hence high interest rates from period 2 onwards - plotted in the middle panel - with continued misallocation of inputs depresses investment after an
initial recovery. The result is a second decline in output as discussed in the main text, a shared conclusion across these computational strategies. The right panel plots the path of forecast errors for prices after an uncertainty shock across methods, and the methods providing the most accurate conditional paths of prices after the shock are the models with extra lags of uncertainty or an extra moment in the forecast rule. These computational structures essentially eliminate any mismatch between forecast and realized prices in the wake of an uncertainty shock, and the left panel of the figure reveals that the basic pattern of output after an uncertainty shock in the more parsimonious baseline economy matches the computationally costly extensions quite well.

**Dynamic Accuracy Statistics**

As Den Haan (2010) notes, one-period ahead or “static” metrics like the R² of the forecast regressions reported in Table B1 are not very stringent tests of the accuracy of a forecasting system in a Krusell-Smith approach. Instead, he recommends using “dynamic” forecasts and evaluating these for accuracy. The procedure for producing dynamic forecasts such as these is to use s-period ahead forecasts as an input into s+1-period ahead forecasts, iterating forwards to any desired horizon conditioning only on the realized values of exogenous aggregate series. Forecast errors can accumulate over time in a manner which is obscured by the one-period ahead accuracy metrics. In this section, we discuss the performance of our model using the dynamic forecast accuracy metrics implied by Den Haan.

In our model general equilibrium impacts a firm’s input problem only through the value of the marginal utility price \( p \). Different from Krusell and Smith (1998)’s economy with a representative firm, the forecasted value of aggregate capital \( K \) does not independently influence prices in the economy outside of its influence on the forecast for \( p \). So to fully characterize the accuracy of a forecast rule in our model, it suffices to examine the accuracy of the implied forecasts for the marginal utility or price \( p \). Therefore, Table B2 provides a comprehensive set of Den Haan accuracy statistics for \( p \) at various horizons. Each block of rows in Table B2 reports the mean and maximum errors in the dynamic forecasts in each of our four computational versions of the model: our baseline, an economy with extra lags in the forecast rule, an economy with an extra moment in the forecast rule, and a version of our baseline with nonconvexified market clearing. Each column reports the errors at a different set of horizons, starting at 3 years (12 periods ahead given the quarterly periodicity of the model) and moving to 12 years (48 model periods or quarters).

For concrete interpretation, note that the number at the far left of the top row, 0.63, indicates that the average price forecasting error of firms in this model at a horizon of 12 quarters or 3 years is 0.63%. The magnitude of error is difficult to interpret on its own. However, by our reading, this level of accuracy is in fact comparable to dynamic forecast errors from other exercises solving firm-level investment models with similarly rich aggregate state spaces. See, for example, the computational appendix to Khan and Thomas (2013), reporting dynamic forecast errors of around 0.8% in a model with both productivity and financial shocks. By contrast, note that papers which report somewhat smaller dynamic forecast errors, such as Terry (2015) at around 0.1% for a version of the baseline Khan and Thomas (2008) model, only so for models with a single aggregate shock and univariate optimization problems at the firm level.

Comparing across methods and horizons in Table B2, we see that the forecasting system with an extra lag of uncertainty performs almost uniformly better than the baseline forecasting system, although not by a large margin. This improvement is unsurprising given the elimination of meaningful swings in the price forecasting error in this economy after an uncertainty shock documented in Figure B2. Interestingly, the economy with an extra moment in the forecast rule performs comparably or worse than the baseline economy, depending on the exact metric used. The direct implication is that additional forecast uncertainty from the accumulated forecast errors in the prediction rule for the extra covariance moment itself degrade the forecasting effectiveness of the expanded system. Also, note that although the economy with nonconvexified clearing appears to perform best at various horizons, it is not strictly comparable to the other methods. The use of a less accurate clearing algorithm implies that this model’s price series is less volatile than the price series for the other
computational strategies purely for numerical reasons.

Taken as a whole, we feel that the good performance of the baseline model using the accuracy metrics in Table B2 is strong enough to justify its continued use, especially in light of the almost constant immediate impact of uncertainty on output revealed by Figure B2 across each alternative forecasting strategy and its smaller computational burden.

**Alternatives to Krusell Smith**

We conclude with a discussion of some of the alternative solution methods proposed in the literature for heterogeneous agents business cycle models. Our analysis uses the Khan and Thomas (2008) extension of Krusell and Smith (1998). More precisely, we rely upon an approximation of the full cross-sectional distribution \( \mu \) by the first moment \( K \) as well as log-linear forecast rules for predicting equilibrium prices \( p \) and the evolution of aggregate capital \( K \). The repeated simulation and forecast rule update steps required by this solution algorithm are quite computationally intensive. In principle, use of an alternative technique such as the method of parameterized distributions (Algan, Allais, and Den Haan 2008, 2010) or the explicit aggregation method (Den Haan and Rendahl 2010) might be more computationally convenient and/or yield gains in terms of the internal accuracy of the approximate equilibrium mappings.

However, the results in Terry (2015) suggest that these alternative algorithms in the context of the closely related Khan and Thomas (2008) model do not yield substantial accuracy improvements over the extended Krusell and Smith (1998) approach. By contrast, that paper reports that both the method of parametrized distributions and the explicit aggregation method yield virtually unchanged quantitative implications for the business cycle in the context of the benchmark lumpy capital adjustment model, but that the internal accuracy of the equilibrium mappings for each method are slightly degraded relative to the Krusell and Smith (1998) algorithm. Furthermore, although both alternative methods yield a substantial reduction in model solution time relative to the Krusell and Smith approach, unconditional simulation time for the model remains costly in the case of the explicit aggregation method and in fact increases for the method of parametrized distributions. Simulation time is particularly critical for our computational approach, because the structural estimation strategy we use relies upon calculation of simulated moments capturing the behavior of micro and macro uncertainty. Therefore, an increase in model solution speed without an increase in simulation speed would yield only a small gain in practice.

The results in Terry (2015) are of course specific to the context of the Khan and Thomas (2008) model rather than our baseline model. However, our model environment is quite similar to the Khan and Thomas (2008) structure, differing primarily through the inclusion of second-moment or uncertainty shocks. Based on this similarity, we conclude that our reliance upon the adapted Krusell and Smith (1998) approach is a reasonable choice in the context of our heterogeneous firms model. Terry (2015) also considers the performance of a conceptually distinct solution technique, the projection plus perturbation method proposed by Reiter (2009). That first-order perturbation approach would not capture the impact of variations over time in aggregate uncertainty in our model, since it relies upon a linearization of the model’s equilibrium around a nonstochastic steady state.
C Online Appendix: Simulated of Moments Estimation

In this section we lay out the SMM approach used for the structural estimation of the uncertainty process in the baseline model. We begin by defining the estimator and noting its well known asymptotic properties. We then discuss in more detail the practical choices made in the implementation of the estimation procedure, both empirically and in the model.

Overview of the Estimator

Our dataset $X$ consists of microeconomic and macroeconomic proxies for uncertainty computed from data covering 1972 – 2010. In each year $t$, an observation consists of

$$X_t = (\text{IQR}_t, \lambda_t, \sigma_{t+1,1}, \sigma_{t+1,2}, \sigma_{t+1,3}, \sigma_{t+1,4}).$$

Above, the variable $\text{IQR}_t$ is the year $t$ cross-sectional interquartile range of TFP shocks constructed from the establishment-level data in the Census of Manufactures and the Annual Survey of Manufacturers and plotted on the left hand scale in Figure 3 in the main text. The variable $\sigma_{t,j}$ is the quarterly estimated GARCH(1,1) heteroskedasticity of the series dTFP from John Fernald’s website, i.e. the annualized change in the aggregate quarterly Solow residual, in year $t$ and quarter $j$ in US data. The macroeconomic series of estimated conditional heteroskedasticity is plotted in Figure A2. Stacking the quarterly observations for the macroeconomic uncertainty proxy $\text{IQR}_t$ in the same year within the vector $X_t$ allows us to account for the frequency difference in the data underlying moment calculation.

Our estimator is based on the $r \times 1 = 8 \times 1$ moment vector including the mean, standard deviation, skewness, and serial correlation of the micro and macro components of $X_t$. The parameter vector $\theta$ is the $q \times 1 = 6 \times 1$ vector $(\sigma^A, \sigma^Z, \sigma^L, \sigma^H, \pi^T, \pi^H)^\prime$. Note that since $r > q$, this is overidentified SMM.

We minimize the scalar objective defined from the moments $g_S(\theta)$ delivered by a model simulation with length $ST$, where $T$ is the length of our simulated dataset in years. Let the vector $g(X)$ be the set of moments computed from the data. Then our estimator $\hat{\theta}$ is equal to

$$\hat{\theta} = \arg \min_\theta (g_S(\theta) - g(X))^\prime W (g_S(\theta) - g(X)), \quad (22)$$

where $W = \text{diag}(1/g(X))^2$, an $r \times r$ symmetric matrix. Therefore, in practice to estimate the model we minimize the sum squared percentage deviations of the model and data moments.

Subject to regularity conditions, we have that by standard SMM arguments laid out in, for example, Lee and Ingram (1991), our SMM estimator is consistent, i.e. $\hat{\theta} \rightarrow_p \theta$, where $\theta$ is the population parameter vector. Furthermore, $\hat{\theta}$ is asymptotically normal with

$$\sqrt{T} (\hat{\theta} - \theta) \rightarrow_d N(0, \Omega).$$

The $q \times q$ asymptotic covariance matrix $\Omega$ also follows standard SMM formulas and is given by

$$\Omega = \left(1 + \frac{1}{S}\right) (A'WA)^{-1} A'W\Sigma W A (A'WA)^{-1'}, \quad (23)$$

where $\Sigma$ is the $r \times r$ asymptotic covariance matrix of the moment vector $g(X)$ and $A$ is the $r \times q$ Jacobian of the moments with respect to the parameter vector $\theta$.

Practical Empirical Implementation

Our estimate of the covariance matrix of the moments $\hat{\Sigma}$ allows for arbitrary stationary serial correlation across years in the underlying series $X_t$ using the block bootstrap procedure with random block length due to Politis and Romano (1994). This procedure also allows for an arbitrary stationary covariance structure across microeconomic and macroeconomic
moments across years and within years in our sample of data. We compute 50,000 bootstrap replications and choose a mean block length of $4 \propto T^{1/3}$ years following the discussion in Politis and Romano (1994).

We compute model moments based on a simulation of 5000 quarters, discarding the initial 500 quarters. Given our conversion to annual frequency and the length of our sample, the simulation multiple parameter $S$ in the asymptotic standard errors is given by $S = 1124/38 \approx 29.58$. The resulting inflation in standard errors due to simulation variability is $1 + \frac{1}{S} \approx 1.03$.

To minimize the SMM objective function and compute $\hat{\theta}$ we use particle swarm optimization, which is a robust global stochastic optimization routine. We also numerically differentiate the model moment function $g_S(\theta)$ at the estimated parameters $\hat{\theta}$ to obtain $\hat{A}$ by averaging over forward-difference approximations starting from the estimated parameters $\hat{\theta}$ with step sizes 0.4, 0.5, and 0.6%. With $\hat{\Sigma}$ and $\hat{A}$ in hand, the estimated asymptotic covariance matrix for $\hat{\theta}$ is given by

$$\hat{\Omega} = \left(1 + \frac{1}{S}\right) \left(\hat{A}'W\hat{A}\right)^{-1} \hat{A}'W\hat{\Sigma}W\hat{A} \left(\hat{A}'W\hat{A}\right)^{-1}.'$$

(24)

Note that there are multiple ways in which the moment covariance matrix $\hat{\Sigma}$ could in principle be estimated, so in unreported results we have also considered the calculation of an estimate of $\Sigma$ based on a long simulation of 250 replication economies within the model itself. We then recalculated our moment vector for each economy and used this set of data to compute an alternative estimate $\hat{\Sigma}$. The resulting moment covariance matrix is similar to the estimate we describe above, never changing inference of the underlying structural parameters meaningfully. However, the alternative approach typically increases the precision of our parameter estimates, so for conservatism we chose to rely on the results from our empirical stationary bootstrap procedure.

Measurement Error in the Data

In this subsection we show how we use OLS and IV estimates of the AR coefficients in the TFP forecast regressions to calculate the share of measurement error in $\log(TFP)$ in our Census micro data sample. The resulting estimate is a useful input into our procedure for computing model equivalents of data moments described in the next subsection. Suppose that (4) is observed with error (omitting the $j$ subscripts)

$$\log(z_t^*) = \log(z_t) + \epsilon_t.$$  

If the measurement $(\epsilon_t)$ error is i.i.d, estimating (4) using $\log(z_{t-2}^*)$ to instrument for $\log(z_{t-1}^*)$ we would obtain a consistent estimate for $\rho$ (Call this estimate $\rho_{IV1}$). The OLS estimate for (4) is inconsistent but the bias is a function of the measurement error in TFP:

$$\rho_{OLS} = \frac{1}{\rho_{IV1}} \left(1 + \frac{\sigma_{IV}^2}{\text{var}(\log(z_t))}\right).$$

We therefore use $\rho_{OLS}$ together with $\rho_{IV1}$ to obtain an estimate for $\sqrt{\frac{\sigma_{IV}^2}{\text{var}(\log(z_t))}}$, which is the share of measurement error in total standard deviation of TFP. The results for $\rho_{OLS}$ and $\rho_{IV1}$ are reported in columns (1) and (2) of Table A2 respectively. These estimates yield a measurement error share of 37.4%.

Suppose now that there is some serial correlation in measurement error - which is quite likely given that ASM respondents are shown prior years values when filling in the current year - so that $\text{cov}(\epsilon_t, \epsilon_{t-1}) = \sigma_{t,t-1}$. As before, define $\rho_{IV1}$ to be the estimate for $\rho$ from an IV regression where $\log(z_{t-2}^*)$ is used to instrument for $\log(z_{t-1}^*)$. Define $\rho_{IV2}$ to be the estimate for $\rho$ from an IV regression where $\log(z_{t-3}^*)$ is used to instrument for $\log(z_{t-1}^*)$. Then we can combine the estimates for $\rho_{OLS}$ with $\rho_{IV1}$ and $\rho_{IV2}$ to obtain estimates for measurement error share as well as for $\sigma_{t,t-1}$. For this specification, our estimates imply measurement error share are of 45.4%. 
Practical Model Implementation

The SMM estimation routine requires repeatedly computing the vector of model moments $g_s(\theta)$ given different values of the parameter vector $\theta$. The macroeconomic moments are straightforward to compute, once we have solved the model in general equilibrium following the solution algorithm outlined above. We unconditionally simulate the model for 5000 quarters and discarding the first 500 periods, using the computationally efficient version of our model with lagged uncertainty in the forecast rule and clearing the non-convexified excess demand function. We then run a panel regression of measured simulated TFP on lagged measured values $\hat{zf}_t$ and time and firm fixed effects

$$TFP_s = \log(Y_s) - \frac{1}{3} \log(K_s) - \frac{2}{3} \log(N_s),$$

where $Y_s$, $K_s$, and $N_s$ are the aggregate output, capital, and labor inputs in the economy. The one-third/two-third weighting of capital and labor is consistent with standard values in the macroeconomic literature, and the use of $s$ rather than $t$ is to avoid notational confusion with the annual-frequency microeconomic dispersion series computed below. We compute the annualized change in $dTFP_s = 400(TFP_s - TFP_{s-1})$, estimating the conditional heteroskedasticity for $dTFP$ in each quarter using a GARCH(1,1) model. The mean, standard deviation, skewness, and serial correlation of the resulting series $\sigma_s$ form the macroeconomic block of the model moments $g_s(\theta)$.

The microeconomic moments are based on the cross-sectional dispersion of innovation in regressions of micro-level TFP on lagged values. To run equivalent regressions in the model requires the simulation of a panel of individual firms. We simulate 1000 individual firms for the same 5000 period overall simulation of the model, again discarding the first 500 quarters of data. We must convert the quarterly simulated firm data to annual frequency firms for the same 5000 period overall simulation of the model, again discarding the first 500 periods, using the computationally efficient version of our model with lagged uncertainty in the forecast rule and clearing the non-convexified excess demand function.

We form a quarterly aggregate TFP series equal in quarter $s$ to

$$TFP_s = \log(Y_s) - \frac{1}{3} \log(K_s) - \frac{2}{3} \log(N_s),$$

where $Y_s$, $K_s$, and $N_s$ are the aggregate output, capital, and labor inputs in the economy. The one-third/two-third weighting of capital and labor is consistent with standard values in the macroeconomic literature, and the use of $s$ rather than $t$ is to avoid notational confusion with the annual-frequency microeconomic dispersion series computed below. We compute the annualized change in $dTFP_s = 400(TFP_s - TFP_{s-1})$, estimating the conditional heteroskedasticity for $dTFP$ in each quarter using a GARCH(1,1) model. The mean, standard deviation, skewness, and serial correlation of the resulting series $\sigma_s$ form the macroeconomic block of the model moments $g_s(\theta)$.

The cross-sectional interquartile range $IQR_t$ of the residual innovations to micro-level TFP, $\hat{\xi}_{jt+1}$, forms the micro-level uncertainty proxy for year $t$ in our simulated data. The microeconomic block of the model moments $g_s(\theta)$ is simply given by the mean, standard deviation, skewness, and serial correlation of the series $IQR_t$. Underlying Uncertainty in the Model versus the Uncertainty Proxy

The micro uncertainty proxy used for model estimation, $IQR_t$, differs from the underlying volatility of micro productivity shocks $\sigma^x_\tau$ in the model, in multiple important ways. These differences are crucial to keep in mind when comparing the moderate empirical variability of the uncertainty proxy in Figure 3 with the high estimated variation in underlying volatility upon impact of an uncertainty shock.

First, $IQR_t$ is measured at annual frequency rather than the quarterly frequency of $\sigma^x_\tau$. Second, as in the Census data sample itself, the uncertainty proxy $IQR_t$ is based on establishment-level Solow residuals computed using temporally mismatched data drawn...
from different quarters in the year (labor, capital) or summed throughout the year (output). Third, the series \( IQR_t \) reflects substantial, and empirically plausible, measurement error in the micro data.

Figure C1 plots several decomposed series which unpack the contribution of each of these steps. In the left hand panel, we show a representative 120-quarter period drawn from the unconditional simulation of the model, plotting four series at quarterly frequency.

The series labelled “Quarterly” is the interquartile range of the underlying micro productivity shocks in the model in a given quarter. This reflects the underlying fundamental uncertainty concept in the model.

The second series, labelled “Annual,” is the interquartile range of a normal distribution with standard deviation equal to the standard deviation of the sum of the quarterly productivity shocks within a year. This uncertainty series is constant within the four quarters of a year.

The third series, labelled “Mismatch,” is the interquartile range of measured TFP innovations computed from an annual panel regression on the Solow residuals \( \log(\hat{z}_{jt}) \) from above. This uncertainty series therefore reflects the contribution of mismatch to measured dispersion, but not the contribution of measurement error, and it does not vary within the year.

Finally, the series labeled “Measurement Error” is simply equal to the annual model uncertainty proxy \( IQR_t \) defined above. It is the interquartile range of measured TFP innovation computed from an annual panel regression on the mis-measured Solow residuals \( \log(\hat{z}^*_{jt}) \) and also does not vary within a year.

As is evident from the left hand side of Figure C1, each annual uncertainty measure naturally has a higher level than the quarterly concept. Furthermore, the Mismatch and Measurement Error series fluctuate slightly even if underlying uncertainty does not change, because quarterly productivity shocks throughout a given year lead to input and output responses within firms that are measured at different times and do not wash out in the Solow residual productivity measurement. Finally, the accounting for measurement error of course leads to a higher measured level of dispersion in TFP shocks.

The right hand side of Figure C1 sheds light on the variability of each uncertainty concept relative to its mean level. The first four bar heights are equal to the coefficient of variation of each series from the left hand side, computed over the full unconditional simulation of the model. The fifth bar height reflects the coefficient of variation of our micro uncertainty proxy from the data, i.e. the interquartile range of TFP shocks in the Census of Manufactures sample plotted in Figure 3. Reflecting the large estimated increase in uncertainty upon impact of an uncertainty shock, the quarterly series has a high coefficient of variation of around 75%. The annualization process does little to change this pattern. However, the temporal mismatch of TFP measurement within the year leads to a large reduction in the coefficient of variation to about 28%. Measurement error leads to a higher mean level of the uncertainty proxy and therefore a low coefficient of variation of around the 12% seen in the data.

Evidently, large estimated underlying uncertainty jumps of around 310% upon impact of an uncertainty shock feed into the more muted variability of our uncertainty proxy because of the temporal mismatch of inputs and outputs as well as the large contribution of measurement error.

\[18\text{Note that the use of a constant returns to scale specification of costs shares within the productivity measurement, while underlying productivity in the model feeds into a decreasing returns to scale production function, is also a form of misspecification that contributes to differences between fundamental and underlying shock dispersion. We have found that this distinction makes little difference in practice for the cross-sectional dispersion of measured TFP shocks in the model. Furthermore, the symmetric constant returns treatment of the model and data moments in this respect is appropriate from an econometric standpoint.}\]
Online Appendix: A Representative Firm Model

In this section we lay out the structure of a simple and purely illustrative representative agent and firm model, an adaptation of Brock and Mirman (1972), which we use in the main text to investigate the declining investment path in the medium term after an uncertainty shock. We also lay out a purely illustrative calibration of this model and compute the effect of a capital destruction shock in this framework.

D.1 Model Details

Output is produced using a single capital input $K_t$ with one-period time to build in investment. Production is subject to stochastic productivity shocks $A_t$. A representative household has preferences over consumption in terms of final output. The equilibrium of this neoclassical economy can be derived as a solution to the following planner problem:

$$\max_{\{I_t\}} \mathbb{E} \sum_{t=0}^{\infty} \rho^t \log(C_t)$$

$$C_t + I_t = A_t K_t^\alpha$$

$$K_{t+1} = I_t + (1 - \delta) K_t$$

$$\log(A_{t+1}) = \rho_A \log(A_t) + \sigma_A \varepsilon_{t+1}$$

Model Calibration and Solution

We choose illustrative parameters for our model, which we solve numerically at an annual frequency. The capital depreciation rate is $\delta = 0.09$, the curvature of the aggregate technology is $\alpha = 1/3$, the subjective discount rate is $\rho = 0.95$, the autocorrelation of aggregate productivity in logs is 0.9, and the standard deviation of aggregate productivity shocks is $\sigma_A = 0.025$. We solve the model using discretization of the aggregate capital state with $n_K = 350$ grid points, Howard policy iteration, and a discretization of the aggregate productivity process $A_t$ following Tauchen (1986) with $n_A = 25$ grid points.

A Capital Destruction Shock Experiment

We compute the effects of an unanticipated capital destruction shock in this economy. In broad terms, this experiment is meant to correspond to the medium-term aftermath of an uncertainty shock, after uncertainty levels have subsided to a large degree but the economy is left with a smaller capital stock which must be rebuilt through investment.

To implement this experiment, we simulate 75000 independent economies of 100-year length. Note that the drastically reduced computational burden in this simple model allows for a much larger number of independent simulations than in our full heterogeneous agents structure with uncertainty shocks in the main text. In period 25 in each economy, we impose an exogenous and unanticipated movement to a capital grid point $K^*$, afterwards allowing each economy to evolve normally. We choose the capital grid point $K^*$, lower than the mean of the capital stock in the ergodic distribution of the model, to ensure that on average this shock results in an approximately -10% reduction in aggregate capital relative to its pre-shock mean.

The Figure D1 in this appendix normalizes the shock period to 1 for plotting purposes and shows, for each indicated variable $X$, the percentage deviation of the cross-economy mean of $X$ from it’s pre-shock level, i.e.

$$\hat{X}_t = 100 \log(\bar{X}_t/\bar{X}_0).$$

Here we have that $\bar{X}_t = \frac{1}{75000} \sum_i X_{it}$ is the cross-economy mean of the aggregate $X$ across all economies $i = 1, ..., 75000$ in period $t$. 
Appendix Table A1

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>Establishments (manufacturing) in the sample 2 years or more</th>
<th>Establishments (manufacturing) in the sample 38 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.046*** 0.075*** -0.257* 0.039*** 0.078***</td>
<td>0.075*** -2.542 0.067*** 0.078***</td>
</tr>
<tr>
<td>Mean of Dep. Variable:</td>
<td>0.494 -1.454 18.406 0.384 0.226</td>
<td>0.500 -1.541 22.188 0.392 0.185</td>
</tr>
<tr>
<td>Corr. with GDP growth</td>
<td>-0.366** 0.091 0.029 -0.380** -0.504***</td>
<td>-0.461*** 0.174 -0.013 -0.419*** -0.553***</td>
</tr>
<tr>
<td>Frequency</td>
<td>Annual Annual Annual Annual Annual</td>
<td>Annual Annual Annual Annual Annual</td>
</tr>
<tr>
<td>Underlying sample</td>
<td>1,390,212 1,390,212 1,390,212 1,390,212 1,390,212</td>
<td>130,201 130,201 130,201 130,201 130,201</td>
</tr>
</tbody>
</table>

Notes: Each column reports a time-series OLS regression point estimate (and standard error below in parentheses) of a measure of uncertainty on a recession indicator. The recession indicator is the share of quarters in that year in a recession. Recessions are defined using the NBER data. In the bottom panel we report the mean of the dependent variable and its correlation with real GDP growth. In columns (1) to (5) the sample is the population of manufacturing establishments with 2 years or more of observations in the ASM or CM survey between 1972 and 2009, which contains data on 211,939 establishments across 39 years of data (one more year than the 38 years of regression data since we need lagged TFP to generate a TFP shock measure). In columns (6) to (10) the sample is the population of manufacturing establishments that appear in the 38 years (1972-2009) of the ASM or CM survey, which contains data on 3,449 establishments. In columns (1) and (6) the dependent variable is the cross-sectional standard deviation (S.D.) of the establishment-level ‘shock’ to Total Factor Productivity (TFP). This ‘shock’ is calculated as the residual from the regression of log(TFP) at year t+1 on its lagged value (year t), a full set of year dummies and establishment fixed effects. In columns (2) and (7) we use the cross-sectional skewness of the TFP ‘shock’, in columns (3) and (8) the cross-sectional kurtosis and in columns (4) and (9) the cross-sectional interquartile range of this TFP ‘shock’ as an outlier robust measure. In columns (5) and (10) the dependent variable is the interquartile range of plants’ sales growth. All regressions include a time trend and Census year dummies (for Census year and for 3 lags). Robust standard errors are applied in all columns to control for any potential serial correlation. *** denotes 1% significance, ** 5% significance and * 10% significance. Data available online at http://www.stanford.edu/~nbloom/RUBC.zip
Table A2: Estimates of AR coefficient for TFP Forecast Regressions for Calculation of M.E. Variance

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: log(TFP&lt;sub&gt;t+1&lt;/sub&gt;)</td>
<td>OLS</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td>log(TFP&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>0.795***</td>
<td>0.906***</td>
<td>0.932***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Observations</td>
<td>413,401</td>
<td>413,401</td>
<td>413,401</td>
</tr>
<tr>
<td>Instrument</td>
<td>N/A</td>
<td>log(TFP&lt;sub&gt;t-1&lt;/sub&gt;)</td>
<td>log(TFP&lt;sub&gt;t-2&lt;/sub&gt;)</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is log(TFP) at the establishment level at year t+1. The right hand variable is log(TFP) at t. The regression sample are manufacturing establishments with 25 years or more of observations in the ASM or CM survey between 1972 and 2009, which also have non-missing log(TFP) value for t-1 and t-2. All columns have a full set of establishment and year fixed effects. In column (1) we use OLS regression. In column (2) we instrument for log(TFP<sub>t</sub>) with log(TFP<sub>t-1</sub>), and in column (3) we instrument with log(TFP<sub>t-2</sub>). Standard errors are reported in brackets below every point estimate. *** denotes 1% significance, ** 5% significance and * 10% significance.
### Table B1: Internal Accuracy Statistics for the Approximate Equilibrium Mappings in the Baseline Model

<table>
<thead>
<tr>
<th>Aggregate State Position (A, S, S_{-1})</th>
<th>Capital log(K_{t+1})</th>
<th>Price log(p_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE (%)</td>
<td>R^2</td>
</tr>
<tr>
<td>(1,0,0)</td>
<td>0.38</td>
<td>0.99</td>
</tr>
<tr>
<td>(1,0,1)</td>
<td>0.20</td>
<td>1.00</td>
</tr>
<tr>
<td>(1,1,0)</td>
<td>0.73</td>
<td>0.98</td>
</tr>
<tr>
<td>(1,1,1)</td>
<td>0.49</td>
<td>0.99</td>
</tr>
<tr>
<td>(2,0,0)</td>
<td>0.51</td>
<td>0.99</td>
</tr>
<tr>
<td>(2,0,1)</td>
<td>0.42</td>
<td>0.99</td>
</tr>
<tr>
<td>(2,1,0)</td>
<td>0.34</td>
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<td>(5,1,1)</td>
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**Notes:** Approximate equilibrium forecast mappings for market-clearing prices are \( \log(p_t) = \alpha_p(A_t, S_t, S_{-1}) + \beta_p(A_t, S_t, S_{-1}) \log(K_t) \) and for next period’s capital are \( \log(K_{t+1}) = \alpha_k(A_t, S_t, S_{-1}) + \beta_k(A_t, S_t, S_{-1}) \log(K_t) \). The accuracy statistics above are computed from an unconditional simulation of 5000 quarters of the baseline model, discarding an initial 500 quarters. Each row in the table above displays the performance of the equilibrium mapping conditional upon a subsample of the data characterized by a given triplet of discretized grid points for aggregate productivity \( A_t \), uncertainty \( S_t \), and lagged uncertainty \( S_{-1} \). RMSE represents the root mean squared error of the indicated rule’s static or one-period ahead forecasts, and the \( R^2 \) measure is the standard \( R^2 \) measure computed from the log-linear regression on the appropriate subsample of simulated data.
<table>
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<th>Den Haan Statistics (%)</th>
<th>Stat.</th>
<th>Horizon</th>
<th>3 Years</th>
<th>4 Years</th>
<th>5 Years</th>
<th>12 Years</th>
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<td>Mean</td>
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<td>0.75</td>
<td>0.84</td>
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<td>0.97</td>
<td>1.13</td>
<td>1.25</td>
<td>1.79</td>
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**Notes:** The table above reports the Den Haan (2010) accuracy statistics "Den Haan Statistics" for the forecasting system for market-clearing marginal utility prices $p$. Den Haan statistics are based on forward iteration of the forecasting system for marginal utility out to a pre-specified horizon conditioning only on exogenous processes, substituting $s$-period ahead forecasts as inputs for $(s+1)$-period ahead forecasts and so on. “Mean” and “Max” refer to the mean and maximum of the approximate percentage errors $100 \mid \log(p_t) – \log(p_{DH})\mid$ resulting from these iterative forecasts. The table reports averages of these error statistics using 2,000 forecast start dates in an unconditional simulation of the model. The statistics are calculated for each of four alternative model solution strategies, denoted in the first column.
Average Quarterly GDP Growth Rates

Figure A1: Recessions increase turbulence. Plant rankings in the TFP distribution churn more in recessions.

Notes: Constructed from the Census of Manufacturers and the Annual Survey of Manufacturing establishments, using establishments with 25+ years to address sample selection. Grey shaded columns are share of quarters in recession within a year. Plants’ rank in the TFP distribution is their decile within the industry and year TFP ranking.
Figure A2: Macro volatility calculated from a GARCH(1,1) model estimated from aggregate TFP growth

Notes: The conditional heteroskedasticity series above is estimated using a GARCH(1,1) model for the annualized percent change in the aggregate Solow residual in US quarterly data from 1972-2010, as available on John Fernald’s website (series dTFP). The recession bars refer to standard dates from the NBER Business Cycle Dating Committee’s website.
Figure B1: Convexified excess demand leads to market clearing with essentially arbitrary accuracy

Notes: The figure above plots two series from an unconditional simulation of the model for 5000 quarters, after discarding an initial 500 quarters. The top panel plots the market clearing level of consumption $C_t$ relying on convexification of the excess demand function to clear goods markets. The bottom panel plots the percentage clearing error, i.e. the percentage difference between implied consumption $C_t$ and the reciprocal of the marginal utility price $p_t$ governing firm investment choices, along with the numerical clearing bounds in dotted lines. Over the unconditional simulation of the model, the maximum clearing error is 0.0099%, and the mean error is 0.00032%.
Figure B2: The size of uncertainty's effect varies little with alternative forecasting and market clearing algorithms

Notes: Based on independent simulations of 2500 economies of 100-quarter length. We impose an uncertainty shock in the quarter labelled 1. The baseline economy (x symbols) uses a forecast rule for market clearing prices \( p \), i.e. marginal utility, including lagged uncertainty as a state variable, and market clearing consumption is obtained by clearing a convexified excess demand function. The extra lags, convexified economy (asterisks) includes three lags of uncertainty as state variables and clears markets with a convexified excess demand function. The extra moment, convexified economy (diamonds) adds the cross-sectional covariance of capital and productivity to the forecast rule and clears markets with a convexified excess demand function. The one lag, nonconvexified economy (o symbols) includes lagged uncertainty as a state variable and clears markets with a nonconvexified excess demand function. From the left, we plot the percent deviations of cross-economy average output, consumption, and the price forecasting error (marginal utility price \( p \) – forecast price \( p \)) from quarter 0.
Figure B3: Alternative measures of misallocation comove

**Notes:** Based on independent simulations of 2500 economies of 100-quarter length. We impose an uncertainty shock in the quarter labelled 1, allowing normal evolution of the economy afterwards. In each panel, we plot the average deviation of an alternative proxy for allocative efficiency from its pre-shock value, in the raw units associated with that series in quarter 0. The top row plots the correlation of productivity and inputs: \( \text{corr}(\log k, \log z) \) and \( \text{corr}(\log n, \log z) \). The middle row plots the dispersion of the marginal product of inputs: the standard deviations of \( \text{log} \frac{y}{k} \) and \( \text{log} \frac{y}{n} \). The bottom row plots the covariance of productivity and inputs: \( \text{cov}(\log k, \log z) \) and \( \text{cov}(\log n, \log z) \).
Figure B4: The impact of uncertainty is robust to alternative methods of computing impulse responses

Notes: For the baseline economy (x symbols), we simulate 2500 independent economies of 100-period length. We impose an uncertainty shock in about quarter 50, labeled 1 above. Clockwise from the top left, we plot the percent deviations of cross-economy average output, labor, consumption, and investment from their values in quarter 0. For the simulation differencing case (► symbols), we simulate 2500 pairs of independent economies. The exogenous shocks to all series are identical within a pair. However, we impose an uncertainty shock in one of the two economies within a pair, allowing all other series and outcomes to evolve normally afterwards. In each panel, the figure plots the average across economy pairs of the percent deviation of the indicated series in the shocked economy versus the no shock economy. The parameters are identical across the two impulse responses plotted above, equal to the baseline estimates.
Figure B5: The impact of uncertainty is robust to using a denser grid for productivity shocks

Notes: Based on independent simulations 100-quarter length. For each path, we impose an uncertainty shock in quarter 1. For the baseline economy (x symbols), we use a grid with 5 points for macro productivity A and micro productivity z. For the denser grid economy (► symbols), we use a grid with 9 points for macro productivity A and micro productivity z. The parameterizations are identical in each case and reflect the baseline estimates. Clockwise from the top left, we plot the percent deviations of cross-economy average output, labor, consumption, and investment from their values in quarter 0.
Notes: The IQR series in the left hand panel were drawn from a representative 120-quarter period in the unconditional simulation of the model. “Quarterly” is the underlying quarterly cross-sectional IQR of micro productivity shocks. “Annual” is the IQR of shocks with variance equal to the variance of the sum of the quarterly shocks throughout a given year. “Mismatch” is the IQR of simulated TFP shocks in that year, with TFP computed as in the data using a Solow Residual approach, first-quarter labor, fourth-quarter capital, and the sum of yearly output. “Measurement Error” is the baseline IQR of simulated TFP shocks accounting for measurement error. The bars in the right hand panel are equal to the coefficient of variation of each indicated IQR series from the left hand side, computed over the entire unconditional simulation of the model, along with the coefficient of variation of the annual uncertainty IQR proxy computed from our Census of Manufactures and Annual Survey of Manufactures data from 1972-2010.
Figure D1: The impact of a capital destruction shock in the Brock-Mirman model

Notes: Based on independent simulations of 75,000 economies of 100-year length in the simplified and illustrative representative agent Brock-Mirman model as described in Appendix E. In the year labelled 1 in each economy, we impose a capital destruction shock leading to about a 10% loss of capital on average, allowing the economy to evolve normally thereafter. Clockwise from the top left, we plot the percent deviations of cross-economy average output, capital, consumption, and investment from their values in year 0.