



# Firm dynamics and markup variations: Implications for sunspot equilibria and endogenous economic fluctuations

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## Abstract

This paper analyzes how the interaction between firms' entry-and-exit decisions and variations in competition gives rise to self-fulfilling, expectation-driven fluctuations in aggregate economic activity and in measured total factor productivity (TFP). The analysis is based on a dynamic general equilibrium model in which net business formation is endogenously procyclical and leads to endogenous countercyclical variations in markups. This interaction leads to indeterminacy in which economic fluctuations occur as a result of self-fulfilling shifts in the beliefs of rational forward looking agents. When calibrated with empirically plausible parameter values and driven solely by self-fulfilling shocks to expectations, the model can quantitatively account for the main empirical regularities characterizing postwar U.S. business cycles and for 65% of the fluctuations in measured TFP.

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## 1. Introduction

This paper analyzes how the interaction between firms' entry/exit decisions and variations in competition gives rise to self-fulfilling, expectation-driven fluctuations in aggregate economic activity and in measured total factor productivity (TFP). To this end, I formulate a dynamic general equilibrium model in which fluctuations in the economy can occur, even in the absence of shocks to fundamentals. Consumer sentiment may drive the economy into self-fulfilling, alternating periods

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of contractions and expansions through the effect of consumers' expectations on their aggregate demand. Variations in aggregate demand lead to variations in the number of operating firms, which in turn affect the degree of competition, leading to fluctuations in aggregate economic activity.

Three basic stylized facts motivate this paper: the existence of monopoly power in the U.S. economy; procyclical variations in the number of competitors; and markups being countercyclical and negatively correlated with the number of competitors. To model the interaction between firms' entry/exit decisions and markup variations, I assume that the economy is characterized by the presence of many different sectors. Each sector is comprised of different, monopolistically competitive intermediate firms. Within a given sector, each firm takes into account the effect that the pricing and production of other firms have on the demand for its goods. This leads to the price elasticity of demand for the typical firm being positively related to the number of firms in its sector, and induces the setting of a lower markup in response to an increase in the number of competitors. The number of firms in a sector is determined by the equilibrium condition that all firms earn zero profits in every period. This condition is enforced by firms' decisions to either enter or exit an industry.<sup>1</sup>

The interaction between net business formation and variations in the degree of competition leads to equilibrium indeterminacy, which ensures that a continuum of stationary sunspot equilibria will exist.<sup>2</sup> The intuition I can provide for why sunspot equilibria occur in the model is as follows: suppose that households expect an increase in output. Other things equal, this leads to a rise in the demand for consumption and investment, inducing new profit opportunities, resulting in net firm creation, and thus to a fall in the markup, which acts as a true technology shifter. In this environment, a positive shock to consumers' expectations generates an increase in aggregate economic activity confirming the initial optimistic expectations, until the economy eventually returns to its non-stochastic steady state.

The existence of sunspot equilibria allows me to explore the empirical implications of the model for economic fluctuations. I find that, for empirically plausible parameter values, a calibrated version of the model when solely driven by self-fulfilling shocks to expectations can quantitatively account for the second-moment properties of key macroeconomic variables in the postwar U.S. data. The model is also consistent with the forecastable movement puzzle emphasized by Rotemberg and Woodford [48] and Benhabib and Wen [10]. Finally, the model gives rise to endogenous variations in measured TFP. These are a result of the effects of variations in the number of operating firms on the markup, and they account for 65% of the volatility in measured TFP in postwar U.S. data.

In their seminal papers, Benhabib and Farmer [6] and Farmer and Guo [25] show that, with sufficiently large returns-to-scale, an otherwise standard RBC model can be indeterminate and can also quantitatively account for business cycle fluctuations when driven by sunspot shocks. However, this result requires a degree of externalities (or increasing returns to scale in the variable factors of production) that is inconsistent with empirical estimates. Benhabib and Farmer [7], Benhabib and Nishimura [9], Harrison [33], and Harrison and Weder [34] show that, when more

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<sup>1</sup> The structural model is close to the model in Portier [42], who documents the procyclicality of business formation and the countercyclicality of markups in French data. He studies the response of his model economy to a technology shock and to a government spending shock, showing that variations in the number of firms and their effect on the markup serve as an internal magnification mechanism. Gali and Zilibotti [27] use a similar structure and study its implications for the existence of poverty traps.

<sup>2</sup> That is, stationary, stochastic, rational expectations equilibria triggered by disturbances that are unrelated to uncertainty about economic fundamentals.

than one production sector is incorporated into the one-sector RBC model, the degree of increasing returns-to-scale required to generate multiple equilibria can be substantially reduced. Wen [55] shows that taking into account the effects of capacity utilization makes endogenous cycles occur for empirically plausible values of returns-to-scale. It is important to emphasize that, in contrast to these studies, the entry/exit model has sunspot equilibria occur even though there are no production externalities, and when the production function exhibits constant returns to scale in the variable factors of production (I assume below the presence of fixed cost of operation. This implies that, overall, the production function is characterized by increasing returns to scale).<sup>3</sup>

Several studies have explored the role of variations in monopoly power in giving rise to endogenous cycles. Peck and Shell [41] prove the existence of sunspot equilibria in a pure exchange-economy in which agents have market power in both commodity and security markets. In Gali [26], variations in the composition of aggregate demand lead to variations in markups and can give rise to endogenous fluctuations. In an illuminating analysis, Schmitt-Grohé [50] shows that the tacit collusion model of Rotemberg and Woodford [45], and a variant of Gali [26], may be characterized by an indeterminate equilibrium. Schmitt-Grohé [50] emphasizes that in order for these models to generate endogenous fluctuations with empirical plausible predictions, the markup value must lie in the upper range of the empirical estimates. Chatterjee et al. [16] study an overlapping-generations model with fiat money and no physical capital. In their model, each agent belongs to one of two sectors and can either consume a home good that is not marketable or engage in the production of his sector's good, pay a production cost, forgo the consumption of the home good, and then exchange the same good for the consumption good of the other sector. The authors show that, in the presence of sufficiently strong complementarities between the two sectors and due to the effects of thick markets on markups, multiple steady states can arise. Finally, Dos Santos and Dufourt [23] study a related model in which there is indeterminacy in the equilibrium number of firms that can take any value within a specific interval. This leads to the existence of sunspot equilibria inducing variations in the number of firms which, coupled with a Cournot model, implies that the model generates countercyclical markups. Thus, one of the key differences between that paper and this one is in the type of sunspot equilibria. In Dos Santos and Dufourt [23], the sunspot equilibria arise from the mere fact that the number of firms is not pinned down and they occur when the model exhibits the saddle property. In contrast, in the current paper the sunspot equilibria are a result of the non-stochastic steady state being a sink. The difference in the type of equilibria considered in the two papers has, in addition to the theoretical difference, several empirical implications. I defer the discussion of these empirical differences until Section 4 in which the empirical predictions of my model are presented.

A survey of the literature estimating the level of markups in the U.S. is beyond the scope of this paper. Overall, the estimates of markups in value added data range from 1.2 to 1.4, while the estimated markups in gross output vary between 1.05 and 1.15.<sup>4</sup> Similarly, different studies have addressed the cyclicity of the markup. Among the most prominent studies finding that markups are countercyclical in the U.S. are Bills [11], Rotemberg and Woodford [44], Rotemberg and Woodford [49], and Chevalier et al. [17]. Martins et al. [38] cover different industries in 14 OECD countries and find markups to be countercyclical in 53 of the 56 cases they consider, with statistically significant result in most of these. In addition, these authors conclude that entry

<sup>3</sup> Early examples of models that exhibit sunspot equilibria can be found in Azariadis [1], and Cass and Shell [14]. See Shell [52] and Benhabib and Farmer [8] for two excellent surveys of models with sunspot equilibria.

<sup>4</sup> See, for example, Hall [31], Morrison [39], Norbin [40], Roeger [43], Martins et al. [38], Basu and Fernald [4], and Basu and Fernald [5].

rates have a negative and statistically significant correlation with markups.<sup>5</sup> Bresnahan and Reiss [12] find that increases in the number of producers increases the competitiveness in the markets they analyze. Similarly, Campbell and Hopenhayn [13] provide empirical evidence to support the argument that firms' pricing decisions are affected by the number of competitors they face; they show that markups react negatively to increases in the number of firms.

The procyclicality of the number of firms has been addressed in Chatterjee and Cooper [15] and Devereux et al. [20]; they show that both net business formation and new business incorporations are strongly procyclical.<sup>6</sup> Similarly, Devereux et al. [20] report that the aggregate number of business failures is countercyclical. Direct measures of the number of operating firms in the U.S. economy exist for the years between 1988 and 2003, providing evidence for the procyclicality of the variations in the number of firms.<sup>7</sup> The contemporaneous correlation between the deviations from the HP trend of the number of firms and the deviations from the HP trend of real GDP equals 0.50 and is significant at the 5% level.<sup>8</sup>

One virtue of the model in this paper is that it represents a minimal perturbation of the prototype perfect-competition RBC model; this enables the paper to highlight the specific role of the interaction between variations in the number of firms and variations in degrees of competition with regard to the existence of indeterminacy. However, this simplicity comes at the cost of descriptive realism, and several empirical caveats should be highlighted. First, while the results above suggest that the aggregate number of competitors varies procyclically in the U.S. data, this empirical observation might be driven by only certain industries, implying that a symmetric model is at odds with the data. To address this issue, I assemble a new data set that documents the number of failing firms in the U.S. economy by industry at a yearly frequency between 1956 and 1996.<sup>9</sup> Table 1 reports the point estimator and the significance level of the contemporaneous correlation between the number of failing firms for each of the industries included in the data set and real GDP. While the point estimator differs across industries, all of the industries are characterized by countercyclical failure rates. This suggests that these are a characteristic of most U.S. industries at different aggregation levels. Of the 46 industries included in the data set, 26 are characterized by countercyclical failure rates statistically significant at the 1% level. Twelve industries are characterized by countercyclical failure rates that are statistically significant at the 5% level, while three industries are characterized by countercyclical failure rates that are statistically significant at the 10% level. For the remaining five industries, the point estimator is negative but not significant at the 10% level.

Second, an additional concern is that smaller firms typically make up the majority of entrants and exits. This may imply that variations in their number are potentially less important and that entry rates should be weighted by the size of entrants.<sup>10</sup> However, it is noteworthy that variations in the number of firms are only one of the channels that generate actual changes in

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<sup>5</sup> Donowitz et al. [22] suggest that markups are procyclical. Rotemberg and Woodford [49] highlight some biases in these results, as they use measures of average variable costs and not marginal costs.

<sup>6</sup> The quarterly data in Devereux et al. [20] is between 1958 and 1995 and is based on Dun & Bradstreet's records. It was discontinued in 1995.

<sup>7</sup> The data set that also contains the number of establishments (see below) can be found in <http://www.sba.gov/advo/research/>

<sup>8</sup> All the correlations refer to the correlation between the deviations from the HP trend of two series. Similarly, whenever the paper reports the standard deviation, it is of an HP-filtered series.

<sup>9</sup> Unfortunately, it has not been possible to construct a similar data set for sectorial entry rates.

<sup>10</sup> A time series of variations in the number of firms by size does not exist, to the best of my knowledge.

Table 1  
Correlation between output and number of failures by industry

| Industry                                | SIC code    | Contemporaneous correlation with aggregate output |
|---|-------------|---|
| Mining                                  |             | −0.2642*  |
| Construction                            |             | −0.5224***  |
| Manufacturing                           |             |   |
| <i>Durable goods</i>                    |             |   |
| Lumber and wood products                | (24)        | −0.5045***  |
| Furniture                               | (25)        | −0.5427***  |
| Stone, clay, and glass products         | (32)        | −0.4657***  |
| Iron and Steel Products                 | (33–34)     | −0.5660***  |
| Electrical and electronic equipment     | (36)        | −0.4686***  |
| Transportation equipment                | (37)        | −0.4471***  |
| Motor vehicle equipment                 | (371)       | −0.3902**   |
| Other machinery                         | (38)        | −0.5757***  |
| Misc. industries                        | (39)        | −0.5925***  |
| <i>Nondurable goods</i>                 |             |   |
| Food and Kindred products               | (20)        | −0.3331**   |
| Textile mill products                   | (22)        | −0.4891***  |
| Apparel and other textile products      | (23)        | −0.5116***  |
| Paper and allied products               | (26)        | −0.3731**   |
| Printing and publishing                 | (27)        | −0.5924***  |
| Chemicals and allied products           | (28)        | −0.3217**   |
| Petroleum, coal, and gas products       | (29)        | −0.2069   |
| Rubber and misc. plastic products       | (30)        | −0.2950**   |
| Leather and leather products            | (31)        | −0.4509**   |
| Transportation and public services      |             | −0.4651***  |
| Wholesale trade                         |             |   |
| <i>Durable</i>                          |             |   |
| Furniture and house furnishings         | (502)       | −0.5204***  |
| Lumber and building materials           | (503)       | −0.5060***  |
| Electrical goods                        | (506)       | −0.3602**   |
| Machinery equipment and supplies        | (508)       | −0.5220***  |
| <i>Nondurable</i>                       |             |   |
| Paper and paper products                | (511)       | −0.2191   |
| Apparel and piece goods                 | (513)       | −0.3565**   |
| Groceries and related products          | (514)       | −0.3692**   |
| Farm-product raw materials              | (515)       | −0.1212   |
| Alcoholic beverages                     | (518)       | −0.2899*  |
| <i>Retail</i>                           |             |   |
| Building, farm, and garden stores       | (52)        | −0.6669***  |
| Food Stores                             | (54)        | −0.4394***  |
| Automotive dealers and service stations | (55)        | −0.4995***  |
| Apparel and accessory stores            | (56)        | −0.4384***  |
| Furniture and furnishings stores        | (57)        | −0.6606***  |
| General and other stores                | (59)        | −0.2034   |
| Liquor stores                           | (592)       | −0.3199**   |
| Miscellaneous retail stores             |             |   |
| Book and stationery stores              | (5942–5943) | −0.3636**   |
| Jewelry stores                          | (5944)      | −0.5503***  |
| Fuel and ice dealers                    | (5982)      | −0.4369***  |

Table 1 (continued)

| Industry                              | SIC code | Contemporaneous correlation with aggregate output |
|---------------------------------------|----------|---|
| <i>Service industries</i>             |          |   |
| Hotels                                | (70)     | −0.4496***  |
| Personal services                     | (72)     |   |
| Cleaning, laundry, repairing services | (721)    | −0.4567***  |
| Funeral services                      | (726)    | −0.1647   |
| Other personal services               | (7299)   | −0.3572**   |
| Business services                     | (73)     | −0.2940*  |
| Repair services other than auto       | (76)     | −0.5502***  |

\*, \*\*, \*\*\* significant at 10%, 5%, and 1% level, respectively.

the number of competitors, which from the model's perspective are the driving force. Variations in the number of establishments are an additional channel affecting the number of competitors. The contemporaneous correlation between the number of establishments and real GDP is 0.44 and is significant at the 5% level. Furthermore, at the business cycle frequency, the number of establishments is significantly volatile. The ratio of the standard deviation of the number of establishments to real GDP is 1.3.

Additional information regarding this issue can be obtained from the Business Employment Dynamics (BED) data set, which documents job gains and losses at the establishment level, and at the quarterly frequency, for the period between the third quarter of 1992 and the second quarter of 2005. The job-gains series includes job-gains from either opening or expanding establishments. Similarly, the job-losses series is comprised of job losses from either closing or contracting establishments. These data allow me to analyze the fraction of job gains and losses explained by opening and closing establishments, respectively. This provides additional information as to the empirical significance of new potential competitors. The first row in Table 2 reports the results in the aggregate U.S. data. The first column in Table 2 shows that the average fraction of the quarterly gross job-gains in the U.S. economy explained by opening establishments is 21.86%. Similarly, the second column reports the fraction of the quarterly job-losses in the U.S. economy that is explained by closing establishments to be 21.17%. While the procyclicality in the number of establishments, together with the importance of variations in them for job creation and destruction, suggest that changes in establishment numbers are quite significant and play a potentially important role in affecting measures of competition, one might wonder how important they are at the business cycle frequency. In order to address this issue, I estimate the fraction of cyclical volatility in job gains and losses that is accounted for by cyclical volatility of employment in opening-and-closing establishments. I use two alternative methods. First, I extract the high-frequency component of the different series, removing the trend from each, using the HP filter. Column III reports how much of the cyclical volatility in job gains is explained by opening establishments: the estimate is 33.46%. Similarly, column IV indicates the extent to which cyclical fluctuations in job losses are explained by closing establishments: again, this fraction is estimated to be above 33%.

A potential bias of this method is that not all of the high-frequency fluctuations are directly attributable to the business cycle. In an attempt to address this issue, I project each of the detrended series on a constant, and on current and lagged detrended aggregate GDP. The measure of cyclical volatility is the percent standard deviation of these estimated projections.<sup>11</sup> Column V reports the

<sup>11</sup> This approach was first introduced by Gomme et al. [28].

Table 2  
Fractions of job gains and job losses accounted for by opening and closing of establishments

|                                    | I   | II   | III   | IV   | V  | VI  |
|------------------------------------|---|--|---|--|--|---|
|                                    | Average fraction of job gains accounted for by opening establishments | Average fraction of job losses accounted for by closing establishments | Fraction of cyclical volatility in job gains accounted for by opening establishments—Method I | Fraction of cyclical volatility in job losses accounted for by closing establishments—Method I | Fraction of cyclical volatility in job gains accounted for by opening establishments—Method II | Fraction of cyclical volatility in job losses accounted for by closing establishments—Method II |
| Aggregate U.S. data                | 0.2186  | 0.2117   | 0.3346  | 0.3375   | 0.1807   | 0.2627  |
| Goods producing                    | 0.1834  | 0.1881   | 0.2893  | 0.2311   | 0.1285   | 0.1303  |
| Natural resources and mining       | 0.1882  | 0.1843   | 0.2950  | 0.3240   | 0.2274   | 0.4288  |
| Information                        | 0.2354  | 0.2422   | 0.3279  | 0.3802   | 0.1998   | 0.2949  |
| Financial activities               | 0.2556  | 0.2688   | 0.4418  | 0.4808   | 0.2981   | 0.2828  |
| Professional and business services | 0.2185  | 0.2261   | 0.3134  | 0.2740   | 0.0837   | 0.1063  |
| Leisure and hospitality            | 0.2659  | 0.2219   | 0.3167  | 0.4705   | 0.1785   | 0.4321  |
| Construction                       | 0.2133  | 0.2147   | 0.2585  | 0.2502   | 0.2033   | 0.2523  |
| Manufacturing                      | 0.1487  | 0.1647   | 0.2993  | 0.2180   | 0.0759   | 0.0657  |
| Service-providing                  | 0.2294  | 0.2196   | 0.3632  | 0.3832   | 0.2007   | 0.3021  |
| Wholesale trade                    | 0.2126  | 0.2370   | 0.3509  | 0.3817   | 0.1512   | 0.2910  |
| Retail trade                       | 0.1926  | 0.1748   | 0.3680  | 0.3981   | 0.2434   | 0.3285  |
| Transportation and warehousing     | 0.1880  | 0.2066   | 0.2667  | 0.2316   | 0.1062   | 0.1872  |

fraction of the cyclical fluctuations in job gains that is accounted for by opening establishments; column VI reports the fraction of the cyclical fluctuations in job losses that is accounted for by closing establishments. Both of these are estimated to be around 20%. This data set also provides additional evidence on whether fluctuations in the number of competitors is a characteristic of most U.S. industries, because it provides measures of job gains and losses for different industries. As can be seen from rows two to thirteen of Table 2, figures similar to those obtained for the aggregate economy are obtained for all of the industries. This suggests that opening and closing establishments are of significant empirical relevance.

Another channel that could generate possibly variations in the number of competitors is the variation in the number of franchises. As Lafontaine and Blair [37] show, sales through franchising amounted to more than 13% of real GDP in the 1980s. Thus, this channel is important for the determination of aggregate output. The contemporaneous correlation between the number of franchises and real GDP is positive and equals 0.32, and the ratio of the standard deviation of the number of franchises to real GDP equals 2.8. These estimates suggest that franchises, as well as establishments, are potentially important sources of fluctuation in the number of competitors.

The changes described in these two examples will not be reflected in the data as changes in the number of firms. However, the model treats all three types of changes as equal forces, and should be seen as analyzing variations in the number of overall competitors, not just in the number of firms. While these results should be approached with caution given the level of aggregation, it is encouraging that these various pieces of evidence all point in the same direction: the existence of significant variations in the number of competitors at the business cycle frequency.

The paper continues as follows. Section 2 presents the benchmark model. Section 3 proves the existence of an indeterminate equilibrium. While the benchmark model is useful for intuition, it is too stylized for empirical analysis. Accordingly, in Section 4, I extend the model and use a calibrated version of it to analyze the quantitative effects of sunspots shocks. Section 5 concludes.

## 2. The benchmark model

The economy is inhabited by a continuum of identical households whose mass is normalized to one. The representative household maximizes

$$\max_{C_t, H_t} E_0 \int_0^\infty \left[ \log(C_t) - \theta \frac{H_t^{1+\chi}}{1+\chi} \right] e^{-\rho t} dt \tag{2.1}$$

subject to the capital law of motion

$$\dot{K}_t = (R_t - \delta)K_t + W_t H_t + \Pi_t - C_t, \tag{2.2}$$

where the initial capital stock is given and equals  $K_0$ .  $C_t$  and  $H_t$  denote consumption and hours worked by the household in period  $t$ , respectively.  $\rho \in (0, 1)$  and  $\delta \in (0, 1)$  denote the subjective time discount factor and the depreciation rate of capital, respectively.  $\chi \geq 0$  governs the Frisch labor supply elasticity, and  $\theta > 0$ . The households own the capital stock and take the equilibrium rental rate,  $R_t$ , and the equilibrium wage,  $W_t$ , as given. Finally, the households own the firms and receive their profits,  $\Pi_t$ . The first-order conditions characterizing the optimal solution are

$$\theta C_t H_t^\chi = W_t, \tag{2.3}$$

$$\frac{\dot{C}_t}{C_t} = R_t - \delta - \rho \tag{2.4}$$

and the following transversality condition holds along the equilibrium path

$$\lim_{t \rightarrow \infty} e^{-\rho t} \frac{K_t}{C_t} = 0. \tag{2.5}$$

*2.1. Technology*

The economy is characterized by a continuum of sectors of measure one. In each sector, there is a finite number of intermediate firms.<sup>13</sup> Each intermediate firm produces a differentiated good. These goods are imperfect substitutes in the production of a sectoral good.<sup>14</sup> In turn, the sectoral goods are imperfect substitutes for each other and are aggregated into a final good. It is assumed that the entry and exit of intermediate producers into the existing sectors take place such that a zero-profit condition is satisfied at each period in each sector.

*Final good production:* The final good is produced with a constant-returns-to-scale production function

$$Y_t = \left[ \int_0^1 Q_t(j)^\omega dj \right]^{\frac{1}{\omega}}, \quad \omega \in (0, 1). \tag{2.6}$$

That is, the final good,  $Y_t$ , aggregates a continuum of measure one of sectoral goods, where  $Q_t(j)$  denotes sector  $j$ 's output. The elasticity of substitution between any two different sectoral goods is constant and equals  $\frac{1}{1-\omega}$ . The final good producers behave competitively, and the households buy the final good for both consumption and investment.

*Sectoral good production:* In each of the  $j$  sectors, there are  $N_t > 1$ , firms producing differentiated goods that are aggregated into a sectoral good by a constant elasticity of substitution aggregating function.<sup>15</sup> The output of the  $j$ th sectoral good is given by

$$Q_t(j) = N_t^{1-\frac{1}{\tau}} \left[ \sum_{i=1}^{N_t} x_t(j, i)^\tau \right]^{\frac{1}{\tau}}, \quad \tau \in (0, 1), \tag{2.7}$$

where  $x_t(j, i)$  is the output of the  $i$ th firm in the  $j$ th sector.<sup>16</sup> The elasticity of substitution between any two goods in a sector is constant and equals  $\frac{1}{1-\tau}$ . The market structure of each sector exhibits monopolistic competition; each  $x_t(j, i)$  is produced by one firm that sets the price for its good in order to maximize its profit. Finally, it is assumed that  $\frac{1}{1-\omega} < \frac{1}{1-\tau}$ ; i.e., the elasticity of substitution between any two goods within a sector is higher than the elasticity of substitution across sectors.

*Intermediate goods production:* Each intermediate good,  $x_t(j, i)$ , is produced using capital,  $k_t(j, i)$ , and labor,  $h_t(j, i)$ , with the following production function:

$$x_t(j, i) = k_t(j, i)^\alpha l_t(j, i)^{1-\alpha} - \phi, \tag{2.8}$$

<sup>12</sup> To simplify the analytical analysis, the model is written in continuous time. For the calibration exercise, I switch to a discrete time model.

<sup>13</sup> A similar setup appears in Rotemberg and Woodford [45].

<sup>14</sup> See the working paper version for an analysis of a model in which the monopolists produce a homogenous good.

<sup>15</sup> Contrary to the measure of sectors, which is constant, the number of firms may vary across periods.

<sup>16</sup> The term  $N^{1-\frac{1}{\tau}}$  in (2.7) implies that there is no ‘‘variety effect’’ in the model. See the working paper version for an analysis of this effect which only magnifies the results here.

where  $\alpha \in [0, 1]$ . The parameter  $\phi > 0$  represents an “overhead cost” component. In each period, an amount  $\phi$  of the intermediate good is immediately used up, independent of how much output is produced. As in Rotemberg and Woodford [48], the role of this parameter is to allow the model to reproduce the apparent absence of pure profits in the U.S. industries despite the presence of market power.<sup>17</sup>

*Final good firms’ problem:* The final good producer solves a static optimization problem that results in the following conditional demand for the  $j$ th sectoral good:

$$Q_t(j) = \left[ \frac{p_t(j)}{P_t} \right]^{\frac{1}{\omega-1}} Y_t, \tag{2.9}$$

where  $p_t(j)$  is the price index of sector  $j$  at period  $t$  and  $P_t$  is the price of the final good at period  $t$ . As usual,  $P_t$  satisfies

$$P_t = \left[ \int_0^1 p_t(j)^{\frac{\omega}{\omega-1}} dj \right]^{\frac{\omega-1}{\omega}}. \tag{2.10}$$

*The intermediate good producer’s problem:* The conditional demand faced by the producer of each  $x_t(j, i)$  variant is

$$x_t(j, i) = \left[ \frac{p_t(j, i)}{p_t(j)} \right]^{\frac{1}{\tau-1}} \frac{Q_t(j)}{N_t}, \tag{2.11}$$

where  $p_t(j, i)$  is the price of the  $i$ th good in sector  $j$  at period  $t$ . Using (2.9) and (2.11), the conditional demand for good  $x_t(j, i)$  at period  $t$  expressed in terms of the final good is

$$x_t(j, i) = \left[ \frac{p_t(j, i)}{p_t(j)} \right]^{\frac{1}{\tau-1}} \left[ \frac{p_t(j)}{P_t} \right]^{\frac{1}{\omega-1}} \frac{Y_t}{N_t}, \tag{2.12}$$

where the sectoral price at period  $t$ ,  $p_t(j)$  equals

$$p_t(j) = N_t^{\frac{1}{\tau}-1} \left[ \sum_{i=1}^{N_t} p_t(j, i)^{\frac{\tau}{\tau-1}} \right]^{\frac{\tau-1}{\tau}}. \tag{2.13}$$

Given the intermediate good producer’s cost function

$$C^x(W_t, R_t, x_t) = \min_{h_t, k_t} W_t h_t + R_t k_t \quad \text{s.t. } k_t^\alpha h_t^{1-\alpha} = x_t + \phi \tag{2.14}$$

and the demand function in (2.12), the intermediate producer solves its maximization problem.

### 2.2. The elasticity of demand

Dixit and Stiglitz [21] assume that the single firm is small relative to the economy, and therefore it does not take into account its effect on the remaining firms. Following this assumption would imply that the  $x_t(j, i)$  producer has no effect on the sectoral price level,  $p_t(j)$ , and on the aggregate

<sup>17</sup> As Rotemberg and Woodford [45] emphasize, one would assume that in a growing economy along a balanced growth path, the fixed cost also grows at the same rate.

price level,  $P_t$ . It then follows from (2.12) that the  $x_t(j, i)$  producer faces a constant price elasticity of demand

$$\eta_{x(j,i)p(j,i)} = \frac{1}{\tau - 1} < 0 \tag{2.15}$$

implying a constant markup rule

$$\mu = \frac{p_t(j, i)}{MC_t(j, i)} = \frac{1}{\tau}. \tag{2.16}$$

However, as Yang and Heijdra [56] emphasize, the assumption in Dixit and Stiglitz [21] is merely an approximation when the “Dixit–Stiglitz aggregator” is defined over a finite number of goods as in (2.7). In this case, the price elasticity of demand that each firm faces is not constant, but rather is a function of the number of goods. This occurs because each monopolistic producer takes into account the effect it has on the price level.

In the model, there is a continuum of sectors, but within each sector there is a finite number of operating firms. This implies that while each  $x_t(j, i)$  producer does not affect the general price level,  $P_t$ , it does affect the sectoral price level,  $p_t(j)$ . The resulting price elasticity of demand faced by the single firm is therefore a function of the number of firms within a sector,  $N_t$ . In a symmetric equilibrium, it is

$$\eta_{x(j,i)p(j,i)}(N_t) = \frac{1}{\tau - 1} + \left[ \frac{1}{\omega - 1} - \frac{1}{\tau - 1} \right] \frac{1}{N_t} \tag{2.17}$$

implying that an increase in  $N_t$  in sector  $j$  induces the  $x_t(j, i)$  producer to face a more elastic demand curve.<sup>18</sup>

A solution to the monopolistic firm’s problem has to satisfy the condition that marginal revenue equals marginal cost

$$\frac{p_t(j, i)}{MC_t(j, i)} = \mu(N_t) = \frac{(1 - \omega)N_t - (\tau - \omega)}{\tau(1 - \omega)N_t - (\tau - \omega)} > 1. \tag{2.18}$$

Note that the markup function is monotonically decreasing in the number of firms, i.e.

$$\frac{d\mu}{dN} < 0 \tag{2.19}$$

and that

$$\tau\mu(N) > 1. \tag{2.20}$$

<sup>18</sup> Notice that in the case where  $N \rightarrow \infty$ , the resulting price elasticity of demand is the same as in (2.15). In this case, the approach in Dixit and Stiglitz [21], and the approach suggested by Yang and Heijdra [56], coincide. Clearly, this is due to the fact that, in this example, each firm has no actual effect on the sectorial price level because it is of a measure zero within a sector.

<sup>19</sup> As mentioned previously, a necessary condition is that  $\tau > \omega$  (this condition is trivially satisfied in the case of a game of homogenous goods like Cournot, as the elasticity of substitution within a sector,  $\tau$ , is infinite). Assuming to the contrary that  $\omega > \tau$  implies that goods within a sector are less substitutable than goods across sectors. In this case, the price elasticity of demand is decreasing in the number of firms within a sector and sunspot equilibria cannot exist. Similarly, in the knife edge case of  $\omega = \tau$  the price elasticity of demand is constant.

<sup>20</sup> From (2.18) it follows that  $\mu(1) = \frac{1}{\omega}$ ;  $\lim_{N \rightarrow \infty} \mu(N) = \frac{1}{\tau}$ . Since  $\tau > \omega$  and the markup function is monotonically decreasing in  $N$ , the result follows immediately.

The monopolistic firm’s conditional demands for hours worked and capital are then given by

$$\frac{W_t}{p_t(j, i)} = \frac{1}{\mu(N_t)} \left[ (1 - \alpha) \frac{k_t^\alpha h_t^{1-\alpha}}{h_t} \right], \tag{2.21}$$

$$\frac{R_t}{p_t(j, i)} = \frac{1}{\mu(N_t)} \left[ \alpha \frac{k_t^\alpha h_t^{1-\alpha}}{k_t} \right]. \tag{2.22}$$

### 2.3. Symmetric rational expectations equilibrium

As the economy’s technology is symmetric with respect to all intermediate inputs, the paper focuses on symmetric equilibria

$$\forall (j, i) \in [0, 1] \times [1, N_t] : x_t(j, i) = x_t, \quad k_t(j, i) = k_t, \quad h_t(j, i) = h_t, \\ p_t(j, i) = p_t, \quad N_t(j) = N_t.$$

Aggregate capital and aggregate hours are then given by

$$K_t = N_t k_t, \quad H_t = N_t h_t.$$

Finally, in the symmetric equilibrium, a zero-profit condition is imposed in every sector in every period, implying

$$p_t x_t = MC_t(x_t + \phi).$$

In a symmetric equilibrium, the intermediate producer’s output, the number of intermediate producers per sector, and the aggregate final output are given by

$$x_t = \frac{\phi}{\mu(N_t) - 1}, \tag{2.23}$$

$$N_t = K_t^\alpha H_t^{1-\alpha} \left[ \frac{\mu(N_t) - 1}{\mu(N_t) \phi} \right], \tag{2.24}$$

$$Y_t = \frac{1}{\mu(N_t)} K_t^\alpha H_t^{1-\alpha}. \tag{2.25}$$

Rewriting (2.24) as

$$N_t = \left[ \frac{\mu(N_t) - 1}{\phi} \right] Y_t$$

it immediately follows that  $N_t$  is procyclical, implying that  $\mu$  is countercyclical.

*The markup variation effect:* I let  $P_t$  be set as the numeraire and equal to 1. This implies that the price charged by an intermediate producer at a symmetric equilibrium is also 1. By (2.21)–(2.22) and (2.25) the equilibrium rental rate and wage in the economy are then given by

$$R_t = \frac{\alpha}{\mu(N_t)} \frac{1}{K_t} \left[ K_t^\alpha H_t^{1-\alpha} \right] = \alpha \frac{Y_t}{K_t} = \frac{1}{\mu(N_t)} [MPK_t], \tag{2.26}$$

$$W_t = \frac{(1 - \alpha)}{\mu(N_t)} \frac{1}{H_t} \left[ K_t^\alpha H_t^{1-\alpha} \right] = (1 - \alpha) \frac{Y_t}{H_t} = \frac{1}{\mu(N_t)} [MPL_t], \tag{2.27}$$

where  $MPK_t$  and  $MPL_t$  denote the marginal productivities of aggregate capital and labor, respectively. Each  $x_t(j, i)$  firm is a monopolist in the production of its own differentiated product

and faces a downward sloping demand curve. The economy’s structure is such that an increase in  $N_t$  endogenously increases the price elasticity of demand that each producer faces, implying that the size of the price reduction required for selling an additional unit is lower. This increases the marginal revenue productivity of the factors of production.

**3. Analysis of dynamics**

**Proposition 1.** *Let a starred variable denote its value in the steady state. Then, a sufficient condition for the existence of a unique non-stochastic steady state is  $\mu^* < \frac{1}{\alpha}$ .*

In the analysis that follows, I assume that this condition holds (all proofs appear in the Appendix).<sup>21</sup> To simplify notation, denote the log of a variable by a lower case. Then, substituting (2.26) into (2.4), the two following dynamic equations are obtained:

$$\dot{k}_t = e^{y_t - k_t} - e^{c_t - k_t} - \delta, \tag{3.1}$$

$$\dot{c}_t = \alpha e^{y_t - k_t} - \delta - \rho. \tag{3.2}$$

In order to express (3.1) and (3.2) as an autonomous pair of differential equations,  $(y_t - k_t)$  needs to be expressed in terms of  $k_t$  and  $c_t$ . From (2.25) and after substituting (2.27) into (2.3) it follows:

$$y_t = \alpha k_t + (1 - \alpha)h_t - \psi_t, \tag{3.3}$$

$$c_t = \log(1 - \alpha) + y_t - (1 + \chi)h_t. \tag{3.4}$$

These last two equations imply that

$$y_t - k_t = v_0 + v_1 k_t + v_2 c_t + v_3 \psi_t, \tag{3.5}$$

where

$$v_0 = \left[ \frac{1 - \alpha}{\chi + \alpha} \right] \log(1 - \alpha), \quad v_1 = \frac{\chi(\alpha - 1)}{\chi + \alpha}, \quad v_2 = - \left[ \frac{1 - \alpha}{\chi + \alpha} \right], \quad v_3 = - \left[ \frac{\chi + 1}{\chi + \alpha} \right].$$

$\psi_t$  can be defined implicitly as

$$\psi_t = \psi(c_t, k_t) = \alpha k_t - (1 - \alpha)c_t - \alpha \ln \left[ e^{k_t} \tau \left[ \frac{(1 - \alpha)}{e^{\psi_t + c_t}} \right]^{\frac{1 - \alpha}{\alpha}} - A_1 \right] + A_2, \tag{3.6}$$

where  $A_1$  and  $A_2$  are two positive constants. Then (3.1) and (3.2) can be rewritten as

$$\dot{k}_t = e^{v_0 + v_1 k_t + v_2 c_t + v_3 \psi(c_t, k_t)} - e^{c_t - k_t} - \delta, \tag{3.7}$$

$$\dot{c}_t = \alpha e^{v_0 + v_1 k_t + v_2 c_t + v_3 \psi(c_t, k_t)} - \delta - \rho. \tag{3.8}$$

Evaluating these at steady-state values, the trace and the determinant are

$$\text{Trace} = \frac{\delta + \rho}{\alpha} (v_1 + \alpha v_2 + v_3 [\psi_k + \alpha \psi_c]) + \frac{\delta(1 - \alpha) + \rho}{\alpha},$$

$$\text{Det} = [\delta(1 - \alpha) + \rho] \left[ \frac{\delta + \rho}{\alpha} \right] [v_1 + v_2 + v_3(\psi_c + \psi_k)],$$

<sup>21</sup> I show later that  $\alpha$  equals the share of capital income in the economy. This value equals 0.3 in the U.S. Thus, as long as the markup is below 3.33, there exists a unique non-stochastic steady state. Given the estimated value of the markup in the U.S. data, this condition seems to be highly plausible.

where  $\psi_c$  and  $\psi_k$  denote the partial derivatives of  $\psi(k, c)$  with respect to  $c$  and  $k$ , respectively. This leads to the following proposition describing the necessary condition for the existence of indeterminacy.

**Proposition 2.** *A necessary condition for the stable manifold to be of dimension two is*

$$(1 - \alpha)\tau\mu^* - 1 > \chi.$$

In the two-dimensional system there is only one predetermined variable,  $k$ . Hence, it follows that when this necessary condition holds, a unique initial point on the stable manifold cannot be pinned down. This implies that the equilibrium is asymptotically stable. From (2.3) it follows that  $\chi$  governs the slope of the labor supply curve. Incorporating the equilibrium condition for the number of firms (2.24) into the “aggregate labor demand” yields the equilibrium locus of wage–hours combinations. The slope of this locus is given by

$$\frac{\partial w_t}{\partial h_t} = (1 - \alpha)\tau\mu^* - 1.$$

Given Proposition 2, it follows that the necessary condition for the existence of an indeterminate equilibrium is that the aggregate “labor demand” (the equilibrium locus of wage–hours combinations) is upward-sloping and steeper than the labor supply curve. Moreover, it is necessary for the markup to vary in response to variations in the number of operating firms; otherwise,  $\tau\mu^* = 1$  and the necessary condition identified in Proposition 2 cannot be satisfied.

The economic interpretation of this necessary condition turns out to be analogous to the one in the seminal paper of Benhabib and Farmer [6]. However, the source of the indeterminate equilibrium is different in this model. In Benhabib and Farmer [6], the mechanism that leads to the existence of an upward-sloping “aggregate labor demand” and to an indeterminate equilibrium, is the presence of technological externalities.<sup>22</sup> In contrast, in the entry/exit model the mechanism operates through the effects of variations in the number of firms on the price elasticity of demand and thus on the marginal revenue productivity of the factors of production. Specifically, assume that at the beginning of period  $t$  agents become “optimistic” about the equilibrium trajectory of consumption. Leisure being a normal good, this revision in expectations shifts the labor supply curve “inward” because of the decline in the marginal utility of wealth. In order for the expectations to be self-validating, consumption needs to be higher in equilibrium; that occurs only if actual production is higher. If the necessary condition for indeterminacy holds, then the increase in the demand for consumption and investment leads to the entry of new firms, which in turn leads to a fall in the markup.<sup>23</sup> This then leads to an increase in the marginal revenue productivity of labor and thus increases the amount of aggregate labor hired in the economy. Capital being a state variable at the impact period, output must increase leading to an increase in consumption and investment, which in turn increases capital in the next period as well, and confirms the initial

<sup>22</sup> As Benhabib and Farmer [6] show, a different interpretation of their model would be to assume that each firm’s production function is characterized by increasing returns to scale in the variable factors of production.

<sup>23</sup> Assume instead that this extra demand is served by the same number of firms. A constant number of firms implies that the markup is also constant. Since variable profits equal  $(\mu(N) - 1)x$ , and since each firm is selling more, variable profits increase. But, since the fixed cost does not depend on aggregate demand, each firm is making positive profits, thus contradicting the definition of the zero-profit equilibrium. Therefore, an increase in demand must be accompanied by an increase in the number of operating firms and, accordingly, by a decrease in the markup.

expectations of the agents. The economy then converges to its non-stochastic steady state as agents increase their consumption of leisure in subsequent periods.

### 3.1. Implications for productivity measures

Given the expression for aggregate output in (2.25), and letting TFP and the Solow residual (SR) be defined in the conventional way, it turns out that

$$TFP_t = \frac{Y_t}{K_t^\alpha H_t^{1-\alpha}} = \frac{1}{\mu(N_t)}, \quad (3.9)$$

$$SR_t = \widehat{Y}_t - s_k \widehat{K}_t - s_H \widehat{H}_t, \quad (3.10)$$

where a hat denotes the percentage deviations from a variable's trend.  $s_k = \alpha$  and  $s_H = 1 - \alpha$  denote the shares of capital income and labor income in final output, respectively.<sup>24</sup> Then, using (2.25), the SR can be written as

$$SR_t = \widehat{TFP}_t = -\widehat{\mu}_t. \quad (3.11)$$

Because it has been established that the markup is countercyclical, variations in TFP must be endogenously procyclical and inversely related to fluctuations in the markup. The channel is as follows: in the model, a positive shock to consumers' expectations generates new profit opportunities that lead to the entry of firms up until the economy reaches a new zero-profit equilibrium. This process results in a fall in the markup. As in (2.23) the ratio of the fixed cost to the actual sales of the monopolistic is

$$\frac{\phi}{x_t} = \mu(N_t) - 1 \quad (3.12)$$

then an expansion leads to a fall in the ratio of fixed cost to actual sales. The economic reasoning behind the fall in the share of the fixed cost is that, following the entry of firms and the fall in the markup, all the firms need to "break even" and make zero profits in equilibrium. The fall in the markup implies that in order to recover the fixed cost of operation, the oligopolistic producer has to sell a higher quantity; this induces the ratio of fixed cost to actual sales to decrease. Because capital and labor are used for the production of both actual sales and the fixed component, this fall in the share of the fixed component implies that a lower share of resources is used for the production of non-actual sales. As TFP is measured only in terms of actual sales, this effect has the same observable implication as a true positive technology shock.<sup>25</sup>

## 4. The extended model

Once the sunspots model is calibrated, its empirical predictions can be analyzed. As it turns out, the time series it generates captures many of the key empirical regularities that characterize the U.S. business cycle. However, the caveat of this result is that, in order for the sunspot version of the model to generate these time series, the required magnitude of the steady-state markup level has to be at least as large as  $\mu^* = 1.6$  which is higher than most of the current estimates

<sup>24</sup> Since there are zero profits in the model economy, the income shares of the factors of production are equal to the elasticity of output with respect to them.

<sup>25</sup> Obviously the opposite process occurs in the case of a negative expectations shock.

in the U.S. data.<sup>26</sup> This result leads me to consider a revised version of the entry/exit, which is followed by a detailed analysis of its empirical predictions. Specifically, I extend the analysis by (i) modifying the firms’ production function to account for the presence of materials usage in the U.S. economy<sup>27</sup> and by (ii) incorporating capacity utilization.<sup>28</sup> These modifications result in the model giving rise to endogenous cycles whose time series predictions resemble those in the U.S. data, with a steady-state markup value over gross output as low as  $\mu_{go}^* = 1.05$ .<sup>29</sup> This value lies well within the current estimates of the markup level in the U.S. economy. It is important to emphasize that countercyclical markup variations are necessary for the existence of an indeterminate steady state. That is, the mere presence of capacity utilization and materials usage does not give rise to an indeterminate equilibrium.

The assumptions with respect to the population, preferences, final good producers, and sectoral output from Section 2 are retained. The production function of the intermediate firms is characterized by a constant elasticity of substitution between value added and materials and is given by

$$x_t(j, i) + \phi = \left( \sigma \left[ (u_t k_t(j, i))^\alpha h_t(j, i)^{1-\alpha} \right]^{-v} + (1 - \sigma) m_t(j, i)^{-v} \right)^{\frac{-1}{v}}, \tag{4.1}$$

where

$$m_t(j, i) = \left[ \int_0^1 q_t(j)^\rho \right]^{\frac{1}{\rho}} \tag{4.2}$$

and where  $u_t \in (0, 1)$  is the rate of capacity utilization.<sup>30,31</sup> Eqs. (4.1) and (4.2) imply that each firm uses an aggregate of the sectoral goods,  $m(j, i)$ , as an input in its production function, which I interpret as materials usage.<sup>32</sup> Note that the demand for each sectoral good,  $q_t(j)$ , and for each producer’s good,  $x_t(j, i)$ , is composed of the demand from two sources: (i) other monopolistic firms that use these as inputs to their own production, and (ii) the demand of final good producers. The rest of the model remains the same.

In order to better illustrate the impact of materials usage and capacity utilization on the stability of the dynamic system, and thus on the susceptibility of the economy to sunspots fluctuations,

<sup>26</sup> The value of  $\mu^* = 1.6$  is obtained when the calibration in Benhabib and Farmer [6,7] and Wen [55] is assumed:  $\rho = 0.99$ ,  $\alpha = 0.3$ ,  $\delta = 0.025$ ,  $\chi = 0$ .

<sup>27</sup> Estimating the shares of labor and materials in gross output in the U.S. manufacturing sector, I find that the share of materials in gross output is consistently above 50% and that the sum of the shares of labor and materials in gross output is around 85%. I therefore calibrate the share of materials in the model to be 50% of gross output.

<sup>28</sup> As Wen [55] emphasizes, the business cycle literature shows that capacity utilization can greatly amplify business cycle shocks (see the references listed there). Wen [55] shows that the introduction of capacity utilization into the Benhabib and Farmer [6] model makes it more susceptible to multiple equilibria.

<sup>29</sup> Note that in the benchmark model, the markup was set by firms that were using only capital and labor for production. This implied that the markup was set over value added. Given the presence of materials usage in this section, the markup is now set over gross output. In what follows  $\mu_{go}(N_t)$  denotes markup over gross output.

<sup>30</sup> Note that I assume that the firms’ elasticity of substitution between the different sectorial outputs is the same as the consumers’ elasticity of substitution. This implies that none of the results depend on the composition of demand.

<sup>31</sup> I use the Greenwood et al. [29] specification for the capacity utilization process. That is, I assume that the household decides on the optimal utilization of capital and that higher utilization implies higher depreciation rates. The specific functional form is given by  $\delta_t = \frac{1}{\theta} u_t^\theta$ ,  $\theta > 1$ .

<sup>32</sup> Basu [3] and Horvath [36], among others, adopt the same interpretation of materials usage.

I concentrate here on the case where the production function is isoelastic in value added and materials (i.e.,  $v = 0$ ). In the numerical exercise that follows,  $v$  is calibrated to the reported value in U.S. data. It can be shown that a necessary condition for the determinant to be positive is

$$\frac{[1 - \alpha\sigma] T_2 \tau \mu_{go}^*}{\tau \mu_{go}^* (1 - T_2) + T_2} > 1. \tag{4.3}$$

The interpretation for this last condition can be better understood once it becomes apparent that the equilibrium locus of wages–hours has the following slope

$$\frac{\partial w}{\partial h} = \frac{(1 - \alpha)\sigma T_2 (\tau \mu_{go}^*)}{[\tau \mu_{go}^* (1 - T_2) + T_2] - (1 - \sigma) T_2 (\tau \mu_{go}^*)} - 1. \tag{4.4}$$

Then, it follows that (4.3) is satisfied if and only if

$$\frac{dw}{dh} > 0. \tag{4.5}$$

Thus, the interpretation of the necessary condition for the steady state to be indeterminate is the same as discussed in Section 3 and in the seminal paper of Benhabib and Farmer [6]: “aggregate labor demand” must be upward-sloping. It can be shown that in order for  $\frac{dw}{dh} > 0$ , and thus for the economy to be susceptible to sunspot fluctuations, the following condition needs to be satisfied:

$$\left[ 1 - \alpha\sigma + \frac{\alpha\sigma}{\theta} \right] \tau \mu_{go}^* > 1. \tag{4.6}$$

**Proposition 3.** (i) *Relative to the “benchmark model”, the “only materials” model is susceptible to sunspot fluctuations for a wider range of parameter values.* (ii) *Relative to the “benchmark model”, the “only utilization” model is susceptible to sunspot fluctuations for a wider range of parameter values.* (iii) *Relative to the other models, the “materials and utilization” model is susceptible to sunspot fluctuations for a wider range of parameter values.* (iv) *The presence of markup variation is a necessary condition for the existence of an indeterminate steady state. That is, the mere presence of capacity utilization and materials usage does not give rise to an indeterminate equilibrium.*

Moreover, one can show that a higher elasticity of substitution, and/or a higher share of materials in gross output makes the economy susceptible to sunspot fluctuations for a wider range of parameters. The economic intuition I can offer for the effect of materials usage is as follows: the production of each firm depends on the output of other firms, which is reflected in its production costs. While labor and capital are always on their respective supply curves, the output of other firms is priced with a markup. This implies that the entry of firms into an existing sector leads to a reduction in the markup charged by each firm in this sector, manifesting itself in lower costs for all of the remaining firms in the economy. Thus, the entry of firms into different sectors creates an added “spillover effect” between sectors that is absent in the value-added analysis. This

<sup>33</sup>Where  $T_1 = \left[ \frac{\theta-1}{\theta-2\sigma} \right]$ ;  $T_2 = \frac{\theta}{\theta-2\sigma}$ .

<sup>34</sup>In the general case in which the elasticity of substitution between value added and materials is different than 1, it can be shown that the necessary condition for the indeterminate equilibrium continues to be  $\frac{dw}{dh} > 0$ .

added “feedback effect” eases the conditions under which the economy is susceptible to sunspot fluctuations, and is stronger, the higher the share of materials in the production of the oligopolistic firm. The economic intuition for the effect of the capacity utilization result is identical to Wen’s [55] result in a version of the Benhabib and Farmer [6] model.

#### 4.1. Time series properties

I rewrite the model in discrete time and solve it by linearizing the equations that characterize the optimal solutions around the steady state. I adopt the calibration of Benhabib and Farmer [6,7] and Wen [55]:  $\rho = 0.99$ ,  $\alpha = 0.3$ ,  $\delta = 0.025$ ,  $\chi = 0$  (indivisible labor). The share of materials in gross output,  $S_M$ , is set to equal 0.5.  $v$  is set such that the elasticity of substitution between value added and materials equals 0.3.<sup>35</sup> The model’s equilibrium conditions imply that  $\hat{n}_t = \frac{1-\tau}{\tau(\mu^*-1)}\hat{y}_t$ . Using the data on the number of firms in the U.S. described earlier, I find that the ratio of the standard deviation of  $\hat{n}_t$  relative to  $\hat{y}_t$  equals 0.5.<sup>36</sup> Given a calibration of the steady-state value of the markup,  $\tau$  can be then found.<sup>37</sup>

I assume that there are no intrinsic shocks to the economy; the only shock to the economy is an i.i.d. sunspot with zero mean introduced into the intertemporal Euler equation, as in Farmer and Guo [25]. As is customary, the standard deviation of the shock is set to match the output volatility in the U.S. data. The evaluation of the model proceeds along the remaining dimensions.

##### 4.1.1. Contemporaneous moments

Table 3 summarizes the business cycle properties of the model. In order to highlight the effect of a steeper labor demand schedule on the results, I present results for two empirically plausible values of the steady-state markup over gross output, 1.05 and 1.10. The model generates business cycle moments that are similar to those of postwar U.S. data. Consumption, investment, and hours worked are procyclical. Investment is more volatile than output, consumption is less volatile than output, and the volatility of hours is similar to that of output. It is interesting to note that the model correctly predicts the high volatility in hours. Hornstein [35] finds that in an economy characterized by monopolistic competition but with constant markups and no entry/exit, the volatility of employment is substantially lower than in U.S. data. This lower volatility occurs because of the dampening effect of overhead costs. While the model here is also characterized by the presence of fixed costs, the effect of the entry/exit on the optimal markup, and thus on the demand for labor, is such that the volatility of hours closely matches the observed value in the data. On the other hand, the consumption series that the entry/exit model generates is characterized by less variability than the one observed in the U.S. data. Note, however, that the version of the model characterized by a higher steady-state markup value improves the result along this dimension. A higher steady-state markup value implies steeper aggregate labor demand; since the “indivisible labor” formulation of Hansen [32] is used in the calibration, the consumption series follows the same patterns as the wage series. A steeper aggregate labor demand implies a wage rate, and therefore consumption,

<sup>35</sup> The quantitative effects of variations in  $v$  are almost insignificant and do not affect the results.

<sup>36</sup> This ratio holds in the data irrespective of whether the data are first differenced or HP filtered.

<sup>37</sup> Nevertheless, one might still be concerned about the robustness of these results. As I discuss in the working paper version, the log linearized markup in a Cournot setup can be expressed as  $\hat{\mu}_t = \left(\frac{1-\mu^*}{1+\mu^*}\right)\hat{y}_t$ , implying that the parameter  $\tau$  does not need to be estimated and that direct calibration of the steady-state markup level determined the movements in the markup. The quantitative results in a Cournot setup are very similar to the results in this model, implying that the results are robust to different values of  $\tau$ . None of the dynamics in this model depend on the value of  $\phi$  and  $\rho$ .

Table 3  
Business cycle moments

|                          | Variable | Std.         | $\rho(Y, X)$ | AC(1) | AC(2) | AC(3) | AR(1) |
|--------------------------|----------|--------------|--------------|-------|-------|-------|-------|
| U.S. data                | y        | 1.57         | 1            | 0.85  | 0.66  | 0.43  | 0.85  |
| Model: $\mu^{go} = 1.05$ |          | 1.57 (0.19)  | 1            | 0.84  | 0.65  | 0.44  | 0.84  |
| Model: $\mu^{go} = 1.10$ |          | 1.57 (0.16)  | 1            | 0.81  | 0.63  | 0.47  | 0.81  |
| U.S. data                | n        | 1.52         | 0.88         | 0.90  | 0.72  | 0.50  | 0.90  |
| Model: $\mu^{go} = 1.05$ |          | 1.55 (0.19)  | 0.99 (0.01)  | 0.84  | 0.65  | 0.44  | 0.84  |
| Model: $\mu^{go} = 1.10$ |          | 1.44 (0.14)  | 0.99 (0.01)  | 0.81  | 0.63  | 0.47  | 0.81  |
| U.S. data                | c        | 1.18         | 0.8          | 0.78  | 0.57  | 0.35  | 0.80  |
| Model: $\mu^{go} = 1.05$ |          | 0.055 (0.19) | 0.48 (0.03)  | 0.93  | 0.81  | 0.63  | 0.93  |
| Model: $\mu^{go} = 1.10$ |          | 0.16 (0.02)  | 0.85 (0.03)  | 0.78  | 0.73  | 0.59  | 0.87  |
| U.S. data                | i        | 7.02         | 0.91         | 0.77  | 0.47  | 0.35  | 0.77  |
| Model: $\mu^{go} = 1.05$ |          | 7.28 (0.19)  | 0.99 (0.01)  | 0.84  | 0.65  | 0.45  | 0.84  |
| Model: $\mu^{go} = 1.10$ |          | 6.85 (0.67)  | 0.99 (0.01)  | 0.81  | 0.63  | 0.47  | 0.81  |
| U.S. data                | y/h      | 0.79         | 0.41         | 0.69  | 0.47  | 0.15  | 0.68  |
| Model: $\mu^{go} = 1.05$ |          | 0.055 (0.19) | 0.48 (0.03)  | 0.93  | 0.81  | 0.63  | 0.93  |
| Model: $\mu^{go} = 1.10$ |          | 0.16 (0.02)  | 0.85 (0.03)  | 0.87  | 0.73  | 0.59  | 0.87  |

Reported moments are means of statistics computed from 250 simulations, each of 132 periods in length. Standard deviations are in parentheses. The U.S. data and the models are HP filtered with a smoothing variable of 1600. The same method has been applied for Tables 4 and 5.

that is more volatile. The low volatility of the consumption series is reflected in the high volatility of the investment time series.<sup>38</sup> Interestingly, in order for the entry/exit model to generate results that closely match the U.S. data, the variation in the markup turns out to be less than 10% of the variation in output. For example, in the simulations, for the case where  $\mu_{go}^* = 1.05$ , the highest value attained for the markup was  $\mu_{go}^{\max} = 1.06$ , while the minimum value was  $\mu_{go}^{\min} = 1.05$ .<sup>39</sup> Finally, note that the model generates productivity that is procyclical and is generated without reliance on any technology shocks. Clearly, while the exact magnitudes of some of the moments generated by the entry/exit model differ from their respective values in the U.S. data, these simulations illustrate that the entry/exit model is able to reproduce many of the key empirical regularities characterizing the U.S. business cycle. Once again, it is important to emphasize that these results are obtained for markup values that are well within the current estimates for U.S. data.

#### 4.1.2. Persistence and auto-covariance functions

Columns 4–7 in Table 3 present the estimation of the auto-correlation functions of several variables of interest in the U.S. data and in the model. As a measure of persistence, I estimate an AR(1) process in the data and from the model's simulation. The model delivers a highly persistent time series, similar to the U.S. data.<sup>40</sup> Moreover, the model's auto-correlation functions closely match those in the U.S. data. Specifically, along the output dimension, the model generates an

<sup>38</sup> See Wen [55] for an illuminating discussion of this issue in his model.

<sup>39</sup> Similarly, in the case where  $\mu_{go}^* = 1.10$ ,  $\mu_{go}^{\max} = 1.13$ , and  $\mu_{go}^{\min} = 1.08$ .

<sup>40</sup> The models based on the benchmark model of Benhabib and Farmer [6] also generate a highly persistent time series.

auto-correlation function and an  $AR(1)$  coefficient that closely resembles those in the U.S. data. The same is true for hours and investment time series. Again, along the consumption dimension, the model performs better, as the steady-state markup increases.

#### 4.1.3. TFP analysis

The properties of the TFP series generated by the model are important for its evaluation, because variations in TFP are the driving force in the standard RBC model that attributes such movements to exogenous changes in technology. Contrary to this, the TFP movements generated by the model here are entirely endogenous. For any value of  $v$ , the resulting “value added TFP” measure is given by

$$\widehat{TFP}_t = \frac{-1}{1 - S_M} \widehat{\mu}_{go,t} + \alpha \widehat{u}_t. \quad (4.7)$$

That is, variations in TFP are an artifact of the variations in markup and utilization rates; it is important to emphasize that the entry/exit dynamics are necessary for these TFP variations to occur in the first place. A variance–covariance decomposition of the TFP variations results in the variance of the markup, and the covariance between the markup and utilization, accounting for 52% of the variation in TFP generated by the model. Overall, the entry/exit model generates a time series of TFP that accounts for 60–73% of these movements in U.S. data while closely matching the auto-correlation functions and the  $AR(1)$  coefficient of the TFP time series in the data.

#### 4.1.4. Forecastable movements

Rotemberg and Woodford [48] emphasize that in U.S. data: (i) changes in output growth rates,  $(\Delta \widehat{y}_t^k)$ , are highly forecastable; (ii) the forecastable changes in hours, investment, and consumption  $(\Delta \widehat{h}_t^k, \Delta \widehat{i}_t^k, \text{ and } \Delta \widehat{c}_t^k, \text{ respectively})$  are strongly positively correlated with forecastable changes in output; (iii) the relative volatility of  $\Delta \widehat{i}_t^k / \Delta \widehat{y}_t^k$  is higher than the relative volatility of  $\Delta \widehat{h}_t^k / \Delta \widehat{y}_t^k$ , and the relative volatility of  $\Delta \widehat{c}_t^k / \Delta \widehat{y}_t^k$  is the lowest of these three. The authors show that these regularities are inconsistent with the prototype RBC model, when driven by permanent technology shocks.<sup>42</sup> A comparison of the empirical evidence, the predictions of the standard RBC model, and the predictions of the entry/exit model follows.<sup>43</sup>

Table 4 reports the predicted and the estimated standard deviation of expected and actual  $k$  quarter changes in output, as well as the ratio between them. As Rotemberg and Woodford [48] show, changes in U.S. output are highly forecastable: more than 50% of actual changes in output are forecastable. In the RBC model, the forecastability of output never exceeds 5%. In the entry/exit model, the forecastability of output growth closely matches the forecastability of output observed in the U.S.

Table 5 reports the correlation between  $\Delta \widehat{y}_t^k$  and  $\Delta \widehat{h}_t^k, \Delta \widehat{i}_t^k, \text{ and } \Delta \widehat{c}_t^k$  as predicted by the model and as estimated in U.S. data. The U.S. data are characterized by a positive correlation between these last three time series and  $\Delta \widehat{y}_t^k$  at all forecasting horizons. The RBC model correctly predicts

<sup>41</sup>As Basu [3] emphasizes, the measurement of TFP movements should be addressed in value added data.

<sup>42</sup>Rotemberg and Woodford [48] analyze an RBC model with government shocks. I analyze the standard RBC model abstracting from government to ease comparison with the previous sections. From now on, I refer to this specific version of the RBC model as simply the “RBC model”.

<sup>43</sup>Benhabib and Wen [10] show that Wen’s [55] model, which is based on technological externalities coupled with capacity utilization, also can account for the forecastable movement puzzle.

Table 4  
Standard deviation of cumulative changes in output

| Horizon                                  | 1      | 2      | 4      | 8      | 12     | 24     |
|--|--------|--------|--------|--------|--------|--------|
| U.S. data                                |        |        |        |        |        |        |
| Forecastable changes in $y$              | 0.0061 | 0.0105 | 0.0186 | 0.0295 | 0.0322 | 0.0305 |
| Actual changes in $y$                    | 0.0107 | 0.0175 | 0.0274 | 0.0379 | 0.0449 | 0.0549 |
| Ratio                                    | 0.5701 | 0.6000 | 0.6788 | 0.7784 | 0.7171 | 0.5556 |
| RBC model                                |        |        |        |        |        |        |
| Forecastable changes in $y$              | 0.0001 | 0.0002 | 0.0005 | 0.0008 | 0.0011 | 0.0016 |
| Actual changes in $y$                    | 0.0100 | 0.0150 | 0.0220 | 0.0320 | 0.0420 | 0.0580 |
| Ratio                                    | 0.0100 | 0.0133 | 0.0227 | 0.0250 | 0.0262 | 0.0276 |
| Entry and exit: $\mu^{\text{EO}} = 1.05$ |        |        |        |        |        |        |
| Forecastable changes in $y$              | 0.0050 | 0.0090 | 0.0180 | 0.0300 | 0.0330 | 0.0160 |
| Actual changes in $y$                    | 0.0090 | 0.0150 | 0.0230 | 0.0330 | 0.0360 | 0.0250 |
| Ratio                                    | 0.5556 | 0.6000 | 0.7826 | 0.9091 | 0.9167 | 0.6400 |
| Entry and exit: $\mu^{\text{EO}} = 1.10$ |        |        |        |        |        |        |
| Forecastable changes in $y$              | 0.0040 | 0.0090 | 0.0190 | 0.0390 | 0.0560 | 0.0900 |
| Actual changes in $y$                    | 0.0100 | 0.0180 | 0.0300 | 0.0480 | 0.0680 | 0.0960 |
| Ratio                                    | 0.4000 | 0.5000 | 0.6333 | 0.8125 | 0.8235 | 0.9375 |

The U.S. data reported is reproduced from Rotemberg and Woodford [48].

the high and positive correlations between  $\Delta\widehat{h}_t^k$ ,  $\Delta\widehat{i}_t^k$  and  $\Delta\widehat{y}_t^k$  but fails with respect to the correlation between  $\Delta\widehat{c}_t^k$  and  $\Delta\widehat{y}_t^k$ . The entry/exit model correctly predicts the high and positive correlation between  $\Delta\widehat{h}_t^k$ ,  $\Delta\widehat{i}_t^k$  and  $\Delta\widehat{y}_t^k$ . Consistent with the data, the entry/exit model predicts a positive correlation between  $\Delta\widehat{c}_t^k$  and  $\Delta\widehat{y}_t^k$ , although with a slightly smaller magnitude. Again, the entry/exit model characterized by a steeper aggregate labor demand curve does better along the consumption dimension.

Table 5 also reports the coefficients from the regression of  $\Delta\widehat{h}_t^k$ ,  $\Delta\widehat{i}_t^k$ , and  $\Delta\widehat{c}_t^k$  on  $\Delta\widehat{y}_t^k$ . As Rotemberg and Woodford [48] show, the U.S. data are characterized by an elasticity of  $\Delta\widehat{c}_t^k$  with respect to  $\Delta\widehat{y}_t^k$ , which is positive and less than 1. The elasticity of  $\Delta\widehat{i}_t^k$  with respect to  $\Delta\widehat{y}_t^k$  is much less smooth, and is estimated to be well above 1. Finally,  $\Delta\widehat{h}_t^k$  responds nearly one to one with  $\Delta\widehat{y}_t^k$ . In contrast, the RBC model predicts (i) negative elasticity of  $\Delta\widehat{c}_t^k$  with respect to  $\Delta\widehat{y}_t^k$ , and (ii) elasticities of  $\Delta\widehat{i}_t^k$  and  $\Delta\widehat{h}_t^k$  with respect to  $\Delta\widehat{y}_t^k$  that are much higher than the one in the data. The entry/exit model predicts a positive elasticity of  $\Delta\widehat{c}_t^k$  with respect to  $\Delta\widehat{y}_t^k$  and predicts  $\Delta\widehat{h}_t^k$  to move almost one to one with  $\Delta\widehat{y}_t^k$ . Regarding the elasticity of  $\Delta\widehat{i}_t^k$  with respect to  $\Delta\widehat{y}_t^k$ , the model predicts this elasticity to be well above 1, but closer to the value estimated in the U.S. data.

As mentioned in the Introduction, Dos Santos and Dufourt [23] study a related model in which the sunspot equilibria arise from the mere fact that the number of firms is not pinned down and they occur when the model exhibits the saddle property. In contrast, in the current paper the sunspot equilibria are a result of the non-stochastic steady state being a sink. The difference in the type of equilibria considered in the two papers has (in addition to the theoretical difference) several empirical implications. First, the fact that the sunspot equilibria in Dos Santos and Dufourt [23] exhibit a saddle path implies that their model generates monotonic convergence after a sunspot shock. This is contrary to the oscillating dynamics that the current paper generates which, as Farmer and Guo [25] and Schmitt-Grohé [51] show, is a key characteristic of the U.S. business cycle. Second, because of their model exhibiting a saddle path, and similarly to the prototype

Table 5  
Correlation and regression coefficients among forecasted changes

| Horizon                                    | 1     | 2     | 4     | 8     | 12    | 24    |
|--|-------|-------|-------|-------|-------|-------|
| U.S. data                                  |       |       |       |       |       |       |
| Correlation between forecasted output and: |       |       |       |       |       |       |
| Forecasted consumption                     | 0.69  | 0.78  | 0.82  | 0.82  | 0.79  | 0.72  |
| Forecasted investment                      | 0.98  | 0.98  | 0.98  | 0.97  | 0.96  | 0.89  |
| Forecasted hours                           | 0.88  | 0.89  | 0.92  | 0.97  | 0.98  | 0.98  |
| Regression on forecasted output of:        |       |       |       |       |       |       |
| Forecasted consumption                     | 0.24  | 0.29  | 0.33  | 0.35  | 0.38  | 0.47  |
| Forecasted investment                      | 3.17  | 3.01  | 2.91  | 2.84  | 2.76  | 2.50  |
| Forecasted hours                           | 1.09  | 1.07  | 0.98  | 0.99  | 1.02  | 1.02  |
| RBC model                                  |       |       |       |       |       |       |
| Correlation between forecasted output and: |       |       |       |       |       |       |
| Forecasted consumption                     | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 |
| Forecasted investment                      | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  |
| Forecasted hours                           | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  |
| Regression on forecasted output of:        |       |       |       |       |       |       |
| Forecasted consumption                     | -6.77 | -6.77 | -6.77 | -6.77 | -6.77 | -6.77 |
| Forecasted investment                      | 29.60 | 29.60 | 29.60 | 29.60 | 29.60 | 29.60 |
| Forecasted hours                           | 7.77  | 7.77  | 7.77  | 7.77  | 7.77  | 7.77  |
| Entry and exit: $\mu^{\text{Eo}} = 1.05$   |       |       |       |       |       |       |
| Correlation between forecasted output and: |       |       |       |       |       |       |
| Forecasted consumption                     | 0.14  | 0.17  | 0.25  | 0.38  | 0.43  | 0.37  |
| Forecasted investment                      | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  |
| Forecasted hours                           | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  |
| Regression on forecasted output of:        |       |       |       |       |       |       |
| Forecasted consumption                     | 0.01  | 0.01  | 0.01  | 0.02  | 0.02  | 0.02  |
| Forecasted investment                      | 4.66  | 4.65  | 4.64  | 4.62  | 4.60  | 4.60  |
| Forecasted hours                           | 0.99  | 0.99  | 0.99  | 0.98  | 0.98  | 0.98  |
| Entry and exit: $\mu^{\text{Eo}} = 1.10$   |       |       |       |       |       |       |
| Correlation between forecasted output and: |       |       |       |       |       |       |
| Forecasted consumption                     | 0.57  | 0.57  | 0.58  | 0.60  | 0.63  | 0.67  |
| Forecasted investment                      | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  |
| Forecasted hours                           | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  | 0.99  |
| Regression on forecasted output of:        |       |       |       |       |       |       |
| Forecasted consumption                     | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  | 0.09  |
| Forecasted investment                      | 4.40  | 4.40  | 4.40  | 4.40  | 4.38  | 4.34  |
| Forecasted hours                           | 0.92  | 0.92  | 0.92  | 0.92  | 0.92  | 0.91  |

The U.S. data reported is reproduced from Rotemberg and Woodford [48].

RBC model (see Cogley and Nason [19]), it cannot generate endogenous persistence. In fact, because the key mechanism for sunspots in Dos Santos and Dufourt [23] is the existence of an interval of the possible number of active firms, they assume that this number follows as an AR1 process in which the innovation is a non-fundamental (sunspot) random variable. The persistence in their model is thus an artifact of the exogenously high persistence they assume in this AR1

process. My model, on the other hand, generates endogenous persistence because the steady state is a sink. Finally, it is worth emphasizing that when their model is simulated with empirically reasonable parameter values, it correctly predicts investment (consumption) to be more (less) volatile than output. Also, as in the U.S. data, consumption, investment and hours are strongly procyclical. However, their model counterfactually generates an hours' volatility which is more than 1.5 times higher than output volatility, and strongly countercyclical wages. Finally, as the mentioned paper does not report its result with respect to fluctuations in TFP and the forecastable movement puzzle, a comparison along these dimensions is not feasible. To summarize, while the two papers are obviously related, they differ (1) with respect to the type of sunspot equilibria that are considered and (2) along some important empirical predictions which are a result of the differences in the type of equilibria.

## 5. Conclusions

This paper formulated an equilibrium business cycle model in which net business formation is endogenously procyclical. The variation in the number of operating firms results in endogenous variation in the markup level along the cycle. The interaction between firms' entry/exit decisions and variation in competition was found to give rise to the existence of indeterminacy in which economic fluctuations occurred as a result of self-fulfilling shifts in the beliefs of rational, forward-looking agents.

The existence of a continuum of stationary sunspot equilibria allowed the exploration of the empirical implications of the model economy for economic fluctuations. Specifically, it was shown that, for empirically plausible parameter values, a calibrated version of the model, driven solely by self-fulfilling shocks to expectations, could quantitatively account for the main empirical regularities characterizing postwar U.S. business cycles. Similarly, although the TFP movements generated by the entry/exit model were entirely endogenous, the model generated a TFP process that could account for 65% of the fluctuations in measured TFP.

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## Appendix A. Proofs

**Proof of Proposition 1.** The zero profits condition implies that  $\frac{1}{\mu(N)} = 1 - \frac{\phi N}{K^\alpha H^{1-\alpha}}$ . One can show that equilibrium hours are constant in any steady state and that equilibrium capital is given by

$K^* = B_1^{\frac{1}{\alpha-1}} \mu(N^*)^{\frac{1}{\alpha-1}}$ , where  $B_1$  is a positive constant. Substituting in  $K^*$  into the zero profit condition yields,  $1 = \mu(N^*) \left[ 1 - B_2 N^* \mu(N^*)^{\frac{\alpha}{1-\alpha}} \right] = F(N^*)$ . It follows that  $\mu(N^*) < \frac{1}{\alpha}$  is a sufficient condition for  $\frac{\partial F}{\partial N^*} < 0$ . Since  $\lim_{N^* \rightarrow \infty} F(N^*) \rightarrow -\infty$ , assuming that  $\frac{1}{1-\omega} \left[ \frac{1}{\omega} \right]^{\frac{\alpha}{1-\alpha}} > \frac{1}{B_2}$  ensures the existence of a unique  $N^*$ . From (2.3) it then follows that there is a unique  $C^*$ .  $\square$

**Proof of Proposition 2.** The stable manifold is of dimension two when there are two eigenvalues with negative real parts. A necessary condition for this to occur is  $\text{Det}(\text{Jacobian}) > 0$ . The sign of the determinant is given by the sign of the following expression  $v_1 + v_2 + v_3(\psi_c + \psi_k)$ . A necessary condition for this expression to be positive, and thus for the determinant to be positive, is  $(1 - \alpha)\tau\mu^* - 1 > \chi$ .  $\square$

**Proof of Proposition 3.** Before addressing the four parts of the proposition I present the necessary conditions for indeterminacy in each of the four models. The necessary condition for indeterminacy in the benchmark model was found to be  $(1 - \alpha)\tau\mu^* > 1$ . Similarly in the “Only Materials Model” the necessary condition is given by  $(1 - \alpha\sigma)\tau\mu_{go}^* > 1$ . In the “Only Utilization Model”, the necessary condition is given by  $\left[ (1 - \alpha) + \frac{\alpha}{\theta} \right] \tau\mu_{va}^* > 1$ , and in the “Utilization and Materials Model” the necessary condition is given by  $\left[ 1 - \alpha\sigma + \frac{\alpha\sigma}{\theta} \right] \tau\mu_{go}^* > 1$ . I now proceed to the formal proof. (i) Follows from the fact that  $\sigma < 1$ ; (ii) from the optimality conditions of the extended model when evaluated at the steady state, it follows that  $\theta = \frac{\rho + \delta}{\delta} > 1$ . Then the proposition is satisfied immediately; (iii) follows immediately from the joint effect identified in (i) and (ii); (iv) in the framework of Dixit and Stiglitz [21],  $\tau\mu^* = 1$ . Then, it follows from the necessary condition in the “Only Utilization Model”, that in order for the non-stochastic steady state to be indeterminate  $\left[ 1 - \alpha\sigma + \frac{\alpha\sigma}{\theta} \right] > 1$  must hold. However, given  $\theta > 1$ ,  $\left[ (1 - \alpha) + \frac{\alpha}{\theta} \right] < 1$  must be true, a condition which cannot be satisfied.  $\square$

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